THE MATHEMATICAL PRINCIPLES OF NATURAL PHILOSOPHY

Sir Isaac Newton, William Davis

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THE
MATHEMATICAL PRINCIPLES
OF
NATURAL PHILOSOPHY.

BY

SIR ISAAC NEWTON.

Translated into English

BY ANDREW MOTTE.

TO WHICH ARE ADDED,

Newton's Syſtem of the World;

A SHORT

Comment on, and Defence of, the Principia,

BY W. EMERSON.

WITH

THE LAWS OF THE MOON'S MOTION

According to Gravity.

BY JOHN MACHIN,

Astron., Prof. at Gresh., and Sec. to the Roy. Soc.

A new Edition,

(With the LIFE of the AUTHOR; and a PORTRAIT, taken from the Buſt in
the Royal Obſervatory at Greenwich)

CAREFULLY REVISED AND CORRECTED BY

W. DAVIS,

Author of the "Treatise on Land Surveying," the "Use of the Globes,"
Editor of the "Mathematical Companion," &c. &c. &c.

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of
THE SYSTEM OF THE WORLD.

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THE

SYSTEM OF THE WORLD.

IT was the antient opinion of not a few, in the earlieſt ages of philoſophy, that the fixed ſtars ſtood immoveable in the higheſt parts of the world; that under the fixed ſtars the planets were carried about the ſun; that the earth, as one of the planets, deſcribed an annual courſe about the ſun, while by a diurnal motion it was in the mean time revolved about its own axis; and that the ſun, as the common fire which ſerved to warm the whole, was fixed in the centre of the univerſe.

This was the philoſophy taught of old by *Philolaus, Ariſtarchus* of *Samos, Plato* in his riper years, and the whole ſect of the *Pythagoreans*; and this was the judgment of *Anaximander*, more antient than any of them; and of that wiſe king of the *Romans, Numa Pompilius*, who, as a ſymbol of the figure of the world with the ſun in the centre, erected a temple in honour of *Veſta*, of a round form, and ordained perpetual fire to be kept in the middle of it.

The *Egyptians* were early obſervers of the heavens; and from them, probably, this philoſophy was ſpread abroad among other nations; for from them it was, and the nations about them, that the *Greeks*, a people of themſelves more addicted to the ſtudy of philology than of nature, derived their firſt, as well as ſoundeſt, notions of philoſophy; and in the veſtal ceremonies we may yet trace the antient ſpirit of the *Egyptians*; for it was their way to deliver their myſteries, that is, their philoſophy of things above the vulgar way of thinking, under the veil of religious rites and hieroglyphic ſymbols.

It is not to be denied but that *Anaxagoras, Democritus*, and others, did now and then ſtart up, who would have it that the earth poſſeſſed the centre of the world, and that the ſtars of all ſorts were revolved towards the weſt about the earth quieſcent in the centre, ſome at a ſwifter, others at a ſlower rate.

However, it was agreed on both ſides that the motions of the celeſtial bodies were performed in ſpaces altogether free and void of reſiſtance. The whim of ſolid orbs was of a later date, introduced by *Eudoxus, Calippus,* and *Ariſtotle;* when the antient philoſophy began to decline, and to give place to the new prevailing fictions of the Greeks.

But, above all things, the phænomena of comets can by no means conſiſt with the notion of ſolid orbs. The *Chaldeans,* the moſt learned aſtronomers of their time, looked upon the comets (which of antient times before had been numbered among the celeſtial bodies) as a particular ſort of planets, which, deſcribing very eccentric orbits, preſented themſelves to our view only by turns, viz. once in a revolution, when they deſcended into the lower parts of their orbits.

And as it was the unavoidable conſequence of the hypotheſis of ſolid orbs; while it prevailed, that the comets ſhould be thruſt down below the moon, ſo no ſooner had the late obſervations of aſtronomers reſtored the comets to their antient places in the higher heavens, but theſe celeſtial ſpaces were at once cleared of the incumbrance of ſolid orbs, which by theſe obſervations were broke into pieces, and diſcarded for ever.

Whence it was that the planets came to be retained within any certain bounds in theſe free ſpaces, and to be drawn off from the rectilinear courſes, which, left to themſelves, they ſhould have purſued, into regular revolutions in curvilinear orbits, are queſtions which we do not know how the antients explained; and probably it was to give ſome ſort of ſatisfaction to this difficulty that ſolid orbs were introduced.

The later philoſophers pretend to account for it either by the action of certain vortices, as *Kepler* and *Des Cartes;* or by ſome other principle of impulſe or attraction, as *Borelli, Hooke,* and others of our nation; for, from the laws of motion, it is moſt certain that theſe effects muſt proceed from the action of ſome force or other.

But our purpoſe is only to trace out the quantity and properties of this force from the phænomena (p. 174, vol. 1 *), and to apply what we diſcover in ſome ſimple caſes as principles, by which, in a mathematical way, we may eſtimate the effects thereof in more involved caſes; for it would be endleſs and impoſſible to bring every particular to direct and immediate obſervation.

We ſaid, *in a mathematical way,* to avoid all queſtions about the nature or quality of this force, which we would not be underſtood to determine by any hypotheſis; and therefore call it by the general name of a centripetal force, as it is a force which is directed towards ſome centre; and as it regards more particularly a body in that centre, we call it circum-ſolar, circum-terreſtrial, circum-jovial; and in like manner in reſpect of other central bodies.

That by means of centripetal forces the planets may be retained in certain orbits, we may eaſily underſtand, if we conſider the motions of projectiles (p. 2, 3, 4, vol. 1); for a ſtone projected is by the preſſure of its own weight forced out of the rectilinear path, which by the projection alone it ſhould have purſued, and made to deſcribe a curve line in the air; and through that crooked way is at laſt brought down to the ground; and the greater the velocity is with which it is projected, the farther it goes before it falls to the earth. We may therefore ſuppoſe the velocity to be ſo increaſed, that it would deſcribe an arc of 1, 2, 5, 10, 100, 1000 miles before it arrived at the earth, till at laſt, exceeding the limits of the earth, it ſhould paſs quite by without touching it.

Let AFB repreſent the ſurface of the earth, C its centre, VD, VE, VF, the curve lines which a body would deſcribe, if projected in an horizontal direction from the top of an high mountain ſucceſſively with more and more velocity (vide vol. 2, p. 180); and, becauſe the celeſtial motions are ſcarcely retarded by the little or no reſiſtance of the ſpaces in which they are performed, to keep up the parity of caſes, let us ſuppoſe either that there is no air about the earth, or at leaſt that it is endowed with little or no power of reſiſting; and for the ſame reaſon that the body projected with a leſs velocity deſcribes the leſſer arc VD, and with a greater velocity the greater arc VE, and, augmenting the velocity, it goes farther and farther to F and G, if the velocity was ſtill more and more augmented, it would reach at laſt quite beyond the circumference of the earth, and return to the mountain from which it was projected.

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* The References are to the New Edition of the Mathematical Principles, &c.

And ſince the areas which by this motion it deſcribes by a radius drawn to the centre of the earth are (by prop. 1, book 1, *Princip. Math.*) proportional to the times in which they are deſcribed, its velocity, when it returns to the mountain, will be no leſs than it was at firſt; and, retaining the ſame velocity, it will deſcribe the ſame curve over and over, by the ſame law.

But if we now imagine bodies to be projected in the directions of lines parallel to the horizon from greater heights, as of 5, 10, 100, 1000, or more miles, or rather as many ſemi-diameters of the earth, thoſe bodies, according to their different velocity, and the different force of gravity in different heights, will deſcribe arcs either concentric with the earth, or variouſly eccentric, and go on revolving through the heavens in thoſe trajectories, juſt as the planets do in their orbs.

As when a ſtone is projected obliquely, that is, any way but in the perpendicular direction, the perpetual deflection thereof towards the earth from the right line in which it was projected is a proof of its gravitation to the earth, no leſs certain than its direct deſcent when only ſuffered to fall freely from reſt; ſo the deviation of bodies moving in free ſpaces from rectilinear paths, and perpetual deflection therefrom towards any place, is a ſure indication of the exiſtence of ſome force which from all quarters impels thoſe bodies towards that place.

And as, from the ſuppoſed exiſtence of gravity, it neceſſarily follows that all bodies about the earth muſt preſs downwards, and therefore muſt either deſcend directly to the earth, if they are let fall from reſt, or at leaſt perpetually deviate from right lines towards the earth, if they are projected ob-

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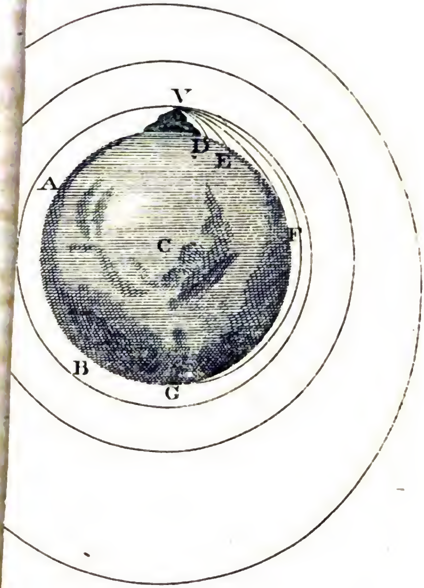

liquely; ſo from the ſuppoſed exiſtence of a force directed to any centre, it will follow, by the like neceſſity, that all bodies upon which this force acts muſt either deſcend directly to that centre, or at leaſt deviate perpetually towards it from right lines, if otherwiſe they ſhould have moved obliquely in theſe right lines.

And how from the motions given we may infer the forces, or from the forces given we may determine the motions, is ſhewn in the two firſt books of our *Principles of Philoſophy.*

If the earth is ſuppoſed to ſtand ſtill, and the fixed ſtars to be revolved in free ſpaces in the ſpace of 24 hours, it is certain the forces by which the fixed ſtars are retained in their orbs are not directed to the earth, but to the centres of the ſeveral orbs, that is, of the ſeveral parallel circles, which the fixed ſtars, declining to one ſide and the other from the equator, deſcribe daily; alſo that by radii drawn to the centres of thoſe orbs the fixed ſtars deſcribe areas exactly proportional to the times of deſcription. Then, becauſe the periodic times are equal (by cor. 3, prop. 4, book 1), it follows that the centripetal forces are as the radii of the ſeveral orbs, and that they will perpetually revolve in the ſame orbs. And the like conſequences may be drawn from the ſuppoſed diurnal motion of the planets.

That forces ſhould be directed to no body on which they phyſically depend, but to innumerable imaginary points in the axis of the earth, is an hypotheſis too incongruous. It is more incongruous ſtill that thoſe forces ſhould increaſe exactly in proportion of the diſtances from this axis; for this is an indication of an increaſe to immenſity, or rather to infinity; whereas the forces of natural things commonly decreaſe in receding from the fountain from which they flow. But, what is yet more abſurd, neither are the areas deſcribed by the ſame ſtar proportional to the times, nor are its revolutions performed in the ſame orb; for as the ſtar recedes from the neighbouring pole, both areas and orb increaſe; and from the increaſe of the area it is demonſtrated that the forces are not directed to the axis of the earth. And this difficulty (cor. 1, prop. 2) ariſes from the twofold motion that is obſerved in the fixed ſtars, one diurnal round the axis of the earth, the other exceedingly ſlow round the axis of the ecliptic. And the explication thereof requires a compoſition of forces ſo perplexed and ſo variable, that it is hardly to be reconciled with any phyſical theory.

That there are centripetal forces actually directed to the bodies of the ſun, of the earth, and other planets, I thus infer.

The moon revolves about our earth, and by radii drawn to its centre (p. 167) deſcribes areas nearly proportional to the times in which they are deſcribed, as is evident from its velocity compared with its apparent diameter; for its motion is ſlower when its diameter is leſs (and therefore its diſtance greater), and its motion is ſwifter when its diameter is greater.

The revolutions of the ſatellites of Jupiter about that planet are more regular (p. 162); for they deſcribe circles concentric with Jupiter by equable motions, as exactly as our ſenſes can diſtinguiſh.

And ſo the ſatellites of Saturn are revolved about this planet with motions nearly (p. 164) circular and equable, ſcarcely diſturbed by any eccentricity hitherto obſerved.

That Venus and Mercury are revolved about the ſun, is demonſtrable from their moon-like appearances (p. 165): when they ſhine with a full face, they are in thoſe parts of their orbs which in reſpect of the earth lie beyond the ſun; when they appear half full, they are in thoſe parts which lie over againſt the ſun; when horned, in thoſe parts which lie between the earth and the ſun; and ſometimes they paſs over the ſun's diſk, when directly interpoſed between the earth and the ſun.

And Venus, with a motion almoſt uniform, deſcribes an orb nearly circular and concentric with the ſun.

But Mercury, with a more eccentric motion, makes remarkable approaches to the ſun, and goes off again by turns; but it is always ſwifter as it is near to the ſun, and therefore by a radius drawn to the ſun ſtill deſcribes areas proportional to the times.

Laſtly, that the earth deſcribes about the ſun, or the ſun about the earth, by a radius from the one to the other, areas exactly proportional to the times, is demonſtrable from the apparent diameter of the ſun compared with its apparent motion.

Theſe are aſtronomical experiments; from which it follows, by prop. 1, 2, 3, in the firſt book of our *Principles*, and their corollaries (p. 167, 168, 171), that there are centripetal forces actually directed (either accurately or without conſiderable error) to the centres of the earth, of Jupiter, of Saturn, and of the ſun. In Mercury, Venus, Mars, and the leſſer planets, where experiments are wanting, the arguments from analogy muſt be allowed in their place.

That thoſe forces (p. 167, 168, 171) decreaſe in the duplicate proportion of the diſtances from the centre of every planet, appears by cor. 6, prop. 4, book 1; for the periodic times of the ſatellites of Jupiter are one to another (p. 162, 163) in the ſeſquiplicate proportion of their diſtances from the centre of this planet.

This proportion has been long ago obſerved in thoſe ſatellites; and Mr. *Flamſted,* who had often meaſured their diſtances from Jupiter by the micrometer, and by the eclipſes of the ſatellites, wrote to me, that it holds to all the accuracy that poſſibly can be diſcerned by our ſenſes. And he ſent me the dimenſions of their orbits taken by the micrometer, and reduced to the mean diſtance of Jupiter from the earth, or from the ſun, together with the times of their revolutions, as follows:

The greateſt elongation of the ſatellites from the centre of Jupiter as ſeen from the ſun.				The periodic times of their revolutions.			
	′	″	″	d.	h.	′	″
1ſt...1	48	or	108	1	18	28	36
2d...3	01	or	181	3	13	17	54
3d...4	46	or	286	7	03	59	36
4th...8	13½	or	493½	16	18	5	13

Whence the ſeſquiplicate proportion may be eaſily ſeen. For example; the $16^{d}.\ 18^{h}.\ 05'\ 13''$ is to the time $1^{d}.\ 18^{h}.\ 28'\ 36''$ as $493\frac{1}{2}'' \times \sqrt{493\frac{1}{2}''}$ to $108'' \times \sqrt{108''}$, neglecting thoſe ſmall fractions which, in obſerving, cannot be certainly determined.

Before the invention of the micrometer, the ſame diſtances were determined in ſemi-diameters of Jupiter thus:

Diſtance of the	1ſt.	2d.	3d.	4th.
By *Galileo*	6	10	16	28
Simon Marius	6	10	16	26
Caſſini	5	8	13	23
Borelli more exactly ..	$5\frac{2}{3}$	$8\frac{2}{3}$	14	$24\frac{2}{3}$

After the invention of the micrometer.

By *Townley*	5,51	8,78	13,47	24,72
Flamſted	5,31	8,85	13,98	24,23
More accurately by the eclipſes .	5,578	8,876	14,159	24,903

And the periodic times of thoſe ſatellites, by the obſervations of Mr. *Flamſted*, are $1^{d}.\ 18^{h}.\ 28'\ 36''$ | $3^{d}.\ 17^{h}.\ 17'\ 54''$ | $7^{d}.\ 3^{h}.\ 59'\ 36''$ | $16^{d}.\ 18^{h}.\ 5'\ 13''$, as above.

And the diſtances thence computed are 5,578 | 8,878 | 14,168 | 24,968, accurately agreeing with the diſtances by obſervation.

Caſſini aſſures us (p. 164, 165) that the ſame proportion is obſerved in the circum-ſaturnal planets. But a longer courſe of obſervations is required before we can have a certain and accurate theory of thoſe planets.

In the circum-ſolar planets, Mercury and Venus, the ſame proportion holds with great accuracy, according to the dimenſions of their orbs, as determined by the obſervations of the beſt aſtronomers.

That Mars is revolved about the ſun is demonſtrated from the phaſes which it ſhews, and the proportion of its apparent diameters (p. 165, 166, and 167); for from its appearing full near conjunction with the ſun, and gibbous in its quadratures, it is certain that it ſurrounds the ſun.

And ſince its diameter appears about five times greater when in oppoſition to the ſun than when in conjunction therewith, and its diſtance from the earth is reciprocally as its apparent diameter, that diſtance will be about five times leſs when in oppoſition to than when in conjunction with the ſun; but in both caſes its diſtance from the ſun will be nearly about the ſame with the diſtance which is inferred from its gibbous appearance in the quadratures. And as it encompaſſes the ſun at almoſt equal diſtances, but in reſpect of the earth is very unequally diſtant, ſo by radii drawn to the ſun it deſcribes areas nearly uniform; but by radii drawn to the earth, it is ſometimes ſwift, ſometimes ſtationary, and ſometimes retrograde.

That Jupiter, in a higher orb than Mars, is likewiſe revolved about the ſun, with a motion nearly equable, as well in diſtance as in the areas deſcribed, I infer thus.

Mr. *Flamſted* aſſured me, by letters, that all the eclipſes of the innermoſt ſatellite which hitherto have been well obſerved do agree with his theory ſo nearly, as never to differ therefrom by two minutes of time; that in the outmoſt the error is little greater; in the outmoſt but one, ſcarcely three times greater; that in the innermoſt but one the difference is indeed much greater, yet ſo as to agree as nearly with his computations as the moon does with the common tables; and that he computes thoſe eclipſes only from the mean motions corrected by the equation of light diſcovered and introduced by Mr. *Romer*. Suppoſing, then, that the theory differs by a leſs error than that of 2′ from the motion of the outmoſt ſatellite as hitherto deſcribed, and taking as the periodic time 16ᵈ. 18ʰ. 5′ 13″ to 2′ in time, ſo is the whole circle or 360° to the arc 1′ 48″, the error of Mr. *Flamſted*'s computation, reduced to the ſatellite's orbit, will be leſs than 1′ 48″; that is, the longitude of the ſatellite, as ſeen from the centre of Jupiter, will be determined with a leſs error than 1′ 48″. But when the ſatellite is in the middle of the ſhadow, that longitude is the ſame with the heliocentric longitude of Jupiter; and, therefore, the hypotheſis which Mr. *Flamſted* follows, viz. the *Copernican*, as improved by *Kepler*, and (as to the motion of

Jupiter) lately corrected by himſelf, rightly repreſents that longitude within a leſs error than 1′ 48″; but by this longitude, together with the geocentric longitude, which is always eaſily found, the diſtance of Jupiter from the ſun is determined; which muſt, therefore, be the very ſame with that which the hypotheſis exhibits. For that greateſt error of 1′ 48″ that can happen in the heliocentric longitude is almoſt inſenſible, and quite to be neglected, and perhaps may ariſe from ſome yet undiſcovered eccentricity of the ſatellite; but ſince both longitude and diſtance are rightly determined, it follows of neceſſity that Jupiter, by radii drawn to the ſun, deſcribes areas ſo conditioned as the hypotheſis requires, that is, proportional to the times.

And the ſame thing may be concluded of Saturn from his ſatellite, by the obſervations of Mr. *Huygens* and Dr. *Halley*; though a longer ſeries of obſervations is yet wanting to confirm the thing, and to bring it under a ſufficiently exact computation.

For if Jupiter was viewed from the ſun, it would never appear retrograde nor ſtationary, as it is ſeen ſometimes from the earth, but always to go forward with a motion nearly uniform (p. 166). And from the very great inequality of its apparent geocentric motion, we infer (by prop. 3, cor. 4) that the force by which Jupiter is turned out of a rectilinear courſe, and made to revolve in an orb, is not directed to the centre of the earth. And the ſame argument holds good in Mars and in Saturn. Another centre of theſe forces is therefore to be looked for (by prop. 2 and 3, and the corollaries of the latter), about which the areas deſcribed by radii intervening may be equable; and that this is the ſun, we have proved already in Mars and Saturn nearly, but accurately enough in Jupiter. It may be alledged that the ſun and planets are impelled by ſome other force equally and in the direction of parallel lines; but by ſuch a force (by cor. 6 of the laws of motion) no change would happen in the ſituation of the planets one to another, nor any ſenſible effect follow: but our buſineſs is with the cauſes of ſenſible effects. Let us, therefore, neglect every ſuch force as imaginary and precarious, and of no uſe in the phænomena of the heavens; and the whole remaining force by which Jupiter is impelled will be directed (by prop. 3, cor. 1) to the centre of the ſun.

The diſtances of the planets from the ſun come out the ſame, whether, with *Tycho*, we place the earth in the centre of the ſyſtem, or the ſun with *Copernicus*: and we have already proved that theſe diſtances are true in Jupiter.

Kepler and *Bullialdus* have, with great care (p. 165), determined the diſtances of the planets from the ſun; and hence it is that their tables agree beſt with the heavens. And in all the planets, in Jupiter and Mars, in Saturn and the Earth, as well as in Venus and Mercury, the cubes of their diſtances are as the ſquares of their periodic times; and therefore (by cor. 6, prop. 4) the centripetal circum-ſolar force throughout all the planetary regions decreaſes in the duplicate proportion of the diſtances from the ſun. In examining this proportion, we are to uſe the mean diſtances, or the tranſverſe ſemi-axes of the orbits (by prop. 15), and to neglect thoſe little fractions, which, in defining the orbits, may have ariſen from the inſenſible errors of obſervation, or may be aſcribed to other cauſes which we ſhall afterwards explain. And thus we ſhall always find the ſaid proportion to hold exactly; for the diſtances of Saturn, Jupiter, Mars, the Earth, Venus, and Mercury, from the ſun, drawn from the obſervations of aſtronomers, are, according to the computation of *Kepler*, as the numbers 951000, 519650, 152350, 100000, 72400, 38806; by the computation of *Bullialdus*, as the numbers 954198, 522520, 152350, 100000, 72398, 38585; and from the periodic times they come out 953806, 520116, 152399, 100000, 72333, 38710. Their diſtances, according to *Kepler* and *Bullialdus*, ſcarcely differ by any ſenſible quantity, and where they differ moſt the diſtances drawn from the periodic times fall in between them.

That the circum-terreſtrial force likewiſe decreaſes in the duplicate proportion of the diſtances, I infer thus.

The mean diſtance of the moon from the centre of the earth, is, in ſemi-diameters of the earth, according to *Ptolomy*, *Kepler* in his *Ephemerides*, *Bullialdus*, *Hevelius*, and

Ricciolus, 59; according to *Flamſted*, $59\frac{1}{3}$; according to *Tycho*, $56\frac{1}{2}$; to *Vendelin*, 60; to *Copernicus*, $60\frac{1}{3}$; to *Kircher*, $62\frac{1}{2}$ (p. 169, 170, 171).

But *Tycho*, and all that follow his tables of refraction, making the refractions of the ſun and moon (altogether againſt the nature of light) to exceed thoſe of the fixed ſtars, and that by about four or five minutes in the horizon, did thereby augment the horizontal parallax of the moon by about the like number of minutes; that is, by about the 12th or 15th part of the whole parallax. Correct this error, and the diſtance will become 60 or 61 ſemi-diameters of the earth, nearly agreeing with what others have determined.

Let us, then, aſſume the mean diſtance of the moon 60 ſemi-diameters of the earth, and its periodic time in reſpect of the fixed ſtars $27^{d}.\ 7^{h}.\ 43'$, as aſtronomers have determined it. And (by cor. 6, prop. 4) a body revolved in our air, near the ſurface of the earth ſuppoſed at reſt, by means of a centripetal force which ſhould be to the ſame force at the diſtance of the moon in the reciprocal duplicate proportion of the diſtances from the centre of the earth, that is, as 3600 to 1, would (ſecluding the reſiſtance of the air) complete a revolution in $1^{h}.\ 24'\ 27''$.

Suppoſe the circumference of the earth to be 123249600 *Paris* feet, as has been determined by the late menſuration of the *French* (vide p. 188); then the ſame body, deprived of its circular motion, and falling by the impulſe of the ſame centripetal force as before, would, in one ſecond of time, deſcribe $15\frac{1}{12}$ *Paris* feet.

This we infer by a calculus formed upon prop. 36, and it agrees with what we obſerve in all bodies about the earth. For by the experiments of pendulums, and a computation raiſed thereon, Mr. *Huygens* has demonſtrated that bodies falling by all that centripetal force with which (of whatever nature it is) they are impelled near the ſurface of the earth, do, in one ſecond of time, deſcribe $15\frac{1}{12}$ *Paris* feet.

But if the earth is ſuppoſed to move, the earth and moon together (by cor. 4 of the laws of motion, and prop. 57) will be revolved about their common centre of gravity. And the moon (by prop. 60) will in the ſame periodic time, $27^{d}.\ 7^{h}.\ 43'$, with the ſame circum-terreſtrial force diminiſhed in the duplicate proportion of the diſtance, deſcribe an orbit whoſe ſemi-diameter is to the ſemi-diameter of the former orbit, that is, to 60 ſemi-diameters of the earth, as the ſum of both the bodies of the earth and moon to the firſt of two mean proportionals between this ſum and the body of the earth; that is, if we ſuppoſe the moon (on account of its mean apparent diameter $31\frac{1}{2}'$) to be about $\frac{1}{42}$ of the earth, as 43 to $\sqrt[3]{42 + \overline{43}|^{2}}$, or as about 128 to 127. And therefore the ſemi-diameter of the orbit, that is, the diſtance between the centres of the moon and earth, will in this caſe be $60\frac{1}{2}$ ſemi-diameters of the earth, almoſt the ſame with that aſſigned by *Copernicus,* which the *Tychonic* obſervations by no means diſprove; and, therefore, the duplicate proportion of the decrement of the force holds good in this diſtance. I have neglected the increment of the orbit which ariſes from the action of the ſun as inconſiderable; but if that is ſubducted, the true diſtance will remain about $60\frac{4}{9}$ ſemi-diameters of the earth.

But farther (p. 167); this proportion of the decrement of the forces is confirmed from the eccentricity of the planets, and the very ſlow motion of their apſes; for (by the corollaries of prop. 45) in no other proportion could the circum-ſolar planets once in every revolution deſcend to their leaſt and once aſcend to their greateſt diſtance from the ſun, and the places of thoſe diſtances remain immoveable. A ſmall error from the duplicate proportion would produce a motion of the apſes conſiderable in every revolution, but in many enormous. But now, after innumerable revolutions, hardly any ſuch motion has been perceived in the orbs of the circum-ſolar planets. Some aſtronomers affirm that there is no ſuch motion; others reckon it no greater than what may eaſily ariſe from the cauſes hereafter to be aſſigned, and is of no moment in the preſent queſtion.

We may even neglect the motion of the moon's apſis (p. 167, 168), which is far greater than in the circum-ſolar planets, amounting in every revolution to three degrees; and from this motion it is demonſtrable that the circum-terreſtrial force decreaſes in no leſs than the duplicate, but far leſs than the triplicate proportion of the diſtance; for if the duplicate proportion was gradually changed into the triplicate, the motion of the apſis would thereby increaſe to infinity; and, therefore, by a very ſmall mutation, would exceed the motion of the moon's apſis. This ſlow motion ariſes from the action of the circum-ſolar force, as we ſhall afterwards explain. But, ſecluding this cauſe, the apſis or apogeon of the moon will be fixed, and the duplicate proportion of the decreaſe of the circum-terreſtrial force in different diſtances from the earth will accurately take place.

Now that this proportion has been eſtabliſhed, we may compare the forces of the ſeveral planets among themſelves (p. 178).

In the mean diſtance of Jupiter from the earth, the greateſt elongation of the outmoſt ſatellite from Jupiter's centre (by the obſervations of Mr. *Flamſted)* is 8′ 13″; and therefore the diſtance of the ſatellite from the centre of Jupiter is to the mean diſtance of Jupiter from the centre of the ſun as 124 to 52012, but to the mean diſtance of Venus from the centre of the ſun as 124 to 7234; and their periodic times are $16\frac{3}{4}^{d}$. and $224\frac{3}{2}^{d}$.; and from hence (according to cor. 2, prop. 4), dividing the diſtances by the ſquares of the times, we infer that the force by which the ſatellite is impelled towards Jupiter is to the force by which Venus is impelled towards the ſun as 442 to 143; and if we diminiſh the force by which the ſatellite is impelled in the duplicate proportion of the diſtance 124 to 7234, we ſhall have the circum-jovial force in the diſtance of Venus from the ſun to the circum-ſolar force by which Venus is impelled as $\frac{13}{100}$ to 143, or as 1 to 1100; wherefore at equal diſtances the circum-ſolar force is 1100 times greater than the circum-jovial.

And, by the like computation, from the periodic time of the ſatellite of Saturn 15^{d}. 22^{h}. and its greateſt elongation from Saturn, while that planet is in its mean diſtance from us, 3′ 20″, it follows that the diſtance of this ſatellite from Saturn's centre is tô the diſtance of Venus from the ſun as 92⅖ to 7234; and from thence that the abſolute circum-ſolar force is 2360 times greater than the abſolute circum-ſaturnal.

From the regularity of the heliocentric and irregularity of the geocentric motions of Venus, of Jupiter, and the other planets, it is evident (by cor. 4, prop. 3) that the circum-terreſtrial force, compared with the circum-ſolar, is very ſmall.

Ricciolus and *Vendelin* have ſeverally tried to determine the ſun's parallax from the moon's dichotomies obſerved by the teleſcope, and they agree that it does not exceed half a minute.

Kepler, from *Tycho*'s obſervations and his own, found the parallax of Mars inſenſible, even in oppoſition to the ſun, when that parallax is ſomething greater than the ſun's.

Flamſted attempted the ſame parallax with the micrometer in the perigeon poſition of Mars, but never found it above 25″; and thence concluded the ſun's parallax at moſt 10″.

Whence it follows that the diſtance of the moon from the earth bears no greater proportion to the diſtance of the earth from the ſun than 29 to 10000; nor to the diſtance of Venus from the ſun than 29 to 7233.

From which diſtances, together with the periodic times, by the method above explained, it is eaſy to infer that the abſolute circum-ſolar force is greater than the abſolute circum-terreſtrial force at leaſt 229400 times.

And though we were only certain, from the obſervations of *Ricciolus* and *Vendelin,* that the ſun's parallax was leſs than half a minute, yet from this it will follow that the abſolute circum-ſolar force exceeds the abſolute circum-terreſtrial force 8500 times.

By the like computations I happened to diſcover an analogy that is obſerved between the forces and the bodies of the planets; but, before I explain this analogy, the apparent diameters of the planets in their mean diſtances from the earth muſt be firſt determined.

Mr. *Flamſted* (p. 163), by the micrometer, meaſured the diameter of Jupiter 40″ or 41″; the diameter of Saturn's ring 50″; and the diameter of the ſun about 32′ 13″ (p. 165).

But the diameter of Saturn is to the diameter of the ring, according to Mr. *Huygens* and Dr. *Halley*, as 4 to 9; according to *Galletius*, as 4 to 10; and according to *Hooke* (by a teleſcope of 60 feet), as 5 to 12. And from the mean proportion, 5 to 12, the diameter of Saturn's body is inferred about 21″.

Such as we have ſaid are the apparent magnitudes; but, becauſe of the unequal refrangibility of light, all lucid points are dilated by the teleſcope, and in the focus of the object-glaſs poſſeſs a circular ſpace whoſe breadth is about the 50th part of the aperture of the glaſs.

It is true, that towards the circumference the light is ſo rare as hardly to move the ſenſe; but towards the middle, where it is of greater denſity, and is ſenſible enough, it makes a ſmall lucid circle, whoſe breadth varies according to the ſplendor of the lucid point, but is generally about the 3d, or 4th, or 5th part of the breadth of the whole.

Let ABD repreſent the circle of the whole light; PQ the ſmall circle of the denſer and clearer light; C the centre of both; CA, CB, ſemi-diameters of the greater circle containing a right angle at C; ACBE the ſquare comprehended under theſe ſemi-diameters; AB the diagonal of that ſquare; EGH an hyperbola with the centre C and aſymptotes CA, CB; PG a perpendicular erected from any point P of the line BC, and meeting the hyperbola in G, and the right lines AB, AE, in K and F: and the denſity of the light in any place P, will, by my computation, be as the line FG, and therefore at the centre infinite, but near the circumference very ſmall. And the whole light within the ſmall circle PQ is to the whole without as the area of the quadrilateral figure CAKP to the triangle PKB. And we are to underſtand the ſmall circle PQ to be there terminated, where FG, the denſity of the light, begins to be leſs that what is required to move the ſenſe.

Hence it was, that, at the diſtance of 191382 feet, a fire of 3 feet in diameter, through a teleſcope of 3 feet, appeared to Mr. *Picart* of 8″ in breadth, when it ſhould have appeared only of 3″ 14‴; and hence it is that the brighter fixed ſtars

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appear through the teleſcope as of 5″ or 6″ in diameter, and that with a good full light; but with a fainter light they appear to run out to a greater breadth. Hence, likewiſe, it was that *Hevelius,* by diminiſhing the aperture of the teleſcope, did cut off a great part of the light towards the circumference, and brought the diſk of the ſtar to be more diſtinctly defined, which, though hereby diminiſhed, did yet appear as of 5″ or 6″ in diameter. But Mr. *Huygens,* only by clouding the eye-glaſs with a little ſmoke, did ſo effectually extinguiſh this ſcattered light, that the fixed ſtars appeared as mere points, void of all ſenſible breadth. Hence alſo it was that Mr. *Huygens,* from the breadth of bodies interpoſed to intercept the whole light of the planets, reckoned their diameters greater than others have meaſured them by the micrometer; for the ſcattered light, which could not be ſeen before for the ſtronger light of the planet, when the planet is hid, appears every way farther ſpread. Laſtly, from hence it is that the planets appear ſo ſmall in the diſk of the ſun, being leſſened by the dilated light. For to *Hevelius, Galletius,* and Dr. *Halley,* Mercury did not ſeem to exceed 12″ or 15″; and Venus appeared to Mr. *Crabtrie* only 1′ 3″; to *Horrox* but 1′ 12″; though by the menſurations of *Hevelius* and *Hugenius* without the ſun's diſk, it ought to have been ſeen at leaſt 1′ 24″. Thus the apparent diameter of the moon, which in 1684, a few days both before and after the ſun's eclipſe, was meaſured at the obſervatory of *Paris* 31′ 30″, in the eclipſe itſelf did not ſeem to exceed 30′ or 30′ 05″; and therefore the diameters of the planets are to be diminiſhed when without the ſun, and to be augmented when within it, by ſome ſeconds. But the errors ſeem to be leſs than uſual in the menſurations that are made by the micrometer. So from the diameter of the ſhadow, determined by the eclipſes of the ſatellites, Mr. *Flamſted* found that the ſemi-diameter of Jupiter was to the greateſt elongation of the outmoſt ſatellite as 1 to 24,903. Wherefore ſince that elongation is 8′ 13″, the diameter of Jupiter will be 39½″; and, rejecting the ſcattered light, the diameter found by the micrometer 40″ or 41″ will be reduced to 39½″; and the diameter of Saturn 21″

is to be diminiſhed by the like correction, and to be reckoned 20″, or ſomething leſs. But (if I am not miſtaken) the diameter of the ſun, becauſe of its ſtronger light, is to be diminiſhed ſomething more, and to be reckoned about 32′, or 32′ 6″.

That bodies ſo different in magnitude ſhould come ſo near to an analogy with their forces, is not without ſome myſtery (p. 180).

It may be that the remoter planets, for want of heat, have not thoſe metallic ſubſtances and ponderous minerals with which our earth abounds; and that the bodies of Venus and Mercury, as they are more expoſed to the ſun's heat, are alſo harder baked, and more compact.

For, from the experiment of the burning-glaſs, we ſee that the heat increaſes with the denſity of light; and this denſity increaſes in the reciprocal duplicate proportion of the diſtance from the ſun; from whence the ſun's heat in Mercury is proved to be ſevenfold its heat in our ſummer ſeaſons. But with this heat our water boils; and thoſe heavy fluids, quickſilver and the ſpirit of vitriol, gently evaporate, as I have tried by the thermometer; and therefore there can be no fluids in Mercury but what are heavy, and able to bear a great heat, and from which ſubſtances of great denſity may be nouriſhed.

And why not, if God has placed different bodies at different diſtances from the ſun, ſo as the denſer bodies always poſſeſs the nearer places, and each body enjoys a degree of heat ſuitable to its condition, and proper for its nouriſhment? From this conſideration it will beſt appear that the weights of all the planets are one to another as their forces.

But I ſhould be glad the diameters of the planets were more accurately meaſured; and that may be done, if a lamp, ſet at a great diſtance, is made to ſhine through a circular hole, and both the hole and the light of the lamp are ſo diminiſhed that the ſpectrum may appear through the teleſcope juſt like the planet, and may be defined by the ſame meaſure: then the diameter of the hole will be to its diſtance from the objective glaſs as the true diameter of the planet to its diſtance from us. The light of the lamp may be diminiſhed by the interpoſition either of pieces of cloth, or of ſmoked glaſs.

Of kin to the analogy we have been deſcribing, there is another obſerved between the forces and the bodies attracted (p. 173, 174, 175). Since the action of the centripetal force upon the planets decreaſes in the duplicate proportion of the diſtance, and the periodic time increaſes in the ſeſquiplicate thereof, it is evident that the actions of the centripetal force, and therefore the periodic times, would be equal in equal planets at equal diſtances from the ſun; and in equal diſtances of unequal planets the total actions of the centripetal force would be as the bodies of the planets; for if the actions were not proportional to the bodies to be moved, they could not equally retract theſe bodies from the tangents of their orbs in equal times: nor could the motions of the ſatellites of Jupiter be ſo regular, if it was not that the circum-ſolar force was equally exerted upon Jupiter and all its ſatellites in proportion of their ſeveral weights. And the ſame thing is to be ſaid of Saturn in reſpect of its ſatellite, and of our earth in reſpect of the moon, as appears from cor. 2 and 3, prop. 65. And, therefore, at equal diſtances, the actions of the centripetal force are equal upon all the planets in proportion of their bodies, or of the quantities of matter in their ſeveral bodies; and for the ſame reaſon muſt be the ſame upon all the particles of the ſame ſize of which the planet is compoſed; for if the action was greater upon ſome ſort of particles than upon others than in proportion to their quantity of matter, it would be alſo greater or leſs upon the whole planets not in proportion of the quantity only, but likewiſe of the ſort of the matter more copiouſly found in one and more ſparingly in another.

In ſuch bodies as are found on our earth of very different ſorts, I examined this analogy with great accuracy (p. 112, 113).

If the action of the circum-terreſtrial force is proportional to the bodies to be moved, it will (by the ſecond law of motion) move them with equal velocity in equal times, and will make all bodies let fall to deſcend through equal ſpaces in equal times, and all bodies hung by equal threads to vibrate in equal times. If the action of the force was greater, the times would be leſs; if that was leſs, theſe would be greater.

But it has been long ago obſerved by others, that (allowance being made for the ſmall reſiſtance of the air) all bodies deſcend through equal ſpaces in equal times; and, by the help of pendulums, that equality of times may be diſtinguiſhed to great exactneſs.

I tried the thing in gold, ſilver, lead, glaſs, ſand, common ſalt, wood, water, and wheat. I provided two equal wooden boxes. I filled the one with wood, and ſuſpended an equal weight of gold (as exactly as I could) in the centre of oſcillation of the other. The boxes, hung by equal threads of 11 feet, made a couple of pendulums perfectly equal in weight and figure, and equally expoſed to the reſiſtance of the air: and, placing the one by the other, I obſerved them to play together forwards and backwards for a long while, with equal vibrations. And therefore (by cor. 1 and 6, prop. 24, book 2) the quantity of matter in the gold was to the quantity of matter in the wood as the action of the motive force upon all the gold to the action of the ſame upon all the wood; that is, as the weight of the one to the weight of the other.

And by theſe experiments, in bodies of the ſame weight, could have diſcovered a difference of matter leſs than the thouſandth part of the whole.

Since the action of the centripetal force upon the bodies attracted is, at equal diſtances, proportional to the quantities of matter in thoſe bodies, reaſon requires that it ſhould be alſo proportional to the quantity of matter in the body attracting.

For all action is mutual, and (p. 14, 27, v. 1, by the third law of motion) makes the bodies mutually to approach one to the other, and therefore muſt be the ſame in both bodies. It is true that we may conſider one body as attracting, another as attracted; but this diſtinction is more mathematical than natural. The attraction is really common of either to other, and therefore of the ſame kind in both.

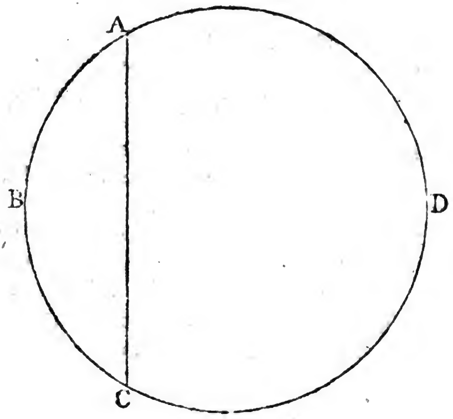
And hence it is that the attractive force is found in both. The ſun attracts Jupiter and the other planets; Jupiter attracts its ſatellites; and, for the ſame reaſon, the ſatellites act as well one upon another as upon Jupiter, and all the planets mutually one upon another.

And though the mutual actions of two planets may be diſtinguiſhed and conſidered as two, by which each attracts the other, yet, as thoſe actions are intermediate, they do not make two but one operation between two terms. Two bodies may be mutually attracted each to the other by the contraction of a cord interpoſed. There is a double cauſe of action, to wit, the diſpoſition of both bodies, as well as a double action in ſo far as the action is conſidered as upon two bodies; but as betwixt two bodies it is but one ſingle one. It is not one action by which the ſun attracts Jupiter, and another by which Jupiter attracts the ſun; but it is one action by which the ſun and Jupiter mutually endeavour to approach each the other. By the action with which the ſun attracts Jupiter, Jupiter and the ſun endeavours to come nearer together (by the third law of motion); and by the action with which Jupiter attracts the ſun, likewiſe Jupiter and the ſun endeavour to come nearer together. But the ſun is not attracted towards Jupiter by a twofold action, nor Jupiter by a twofold action towards the ſun; but it is one ſingle intermediate action, by which both approach nearer together.

Thus iron draws the load-ſtone (p. 27, vol. 1), as well as the load-ſtone draws the iron; for all iron in the neighbourhood of the load-ſtone draws other iron. But the action betwixt the load-ſtone and iron is ſingle, and is conſidered as ſingle by the philoſophers. The action of iron upon the load-ſtone is, indeed, the action of the load-ſtone betwixt itſelf and the iron, by which both endeavour to come nearer together: and ſo it manifeſtly appears; for if you remove the load-ſtone, the whole force of the iron almoſt ceaſes.

In this ſenſe it is that we are to conceive one ſingle action to be exerted betwixt two planets, ariſing from the conſpiring natures of both; and this action ſtanding in the ſame relation to both, if it is proportional to the quantity of matter in the one, it will be alſo proportional to the quantity of matter in the other.

Perhaps it may be objected, that, according to this philoſophy (p. 178), all bodies ſhould mutually attract one another, contrary to the evidence of experiments in terreſtrial bodies; but I anſwer, that the experiments in terreſtrial bodies come to no account; for the attraction of homogeneous ſpheres near their ſurfaces are (by prop 72) as their diameters. Whence a ſphere of one foot in diameter, and of a like nature to the earth, would attract a ſmall body placed near its ſurface with a force 20000000 times leſs than the earth would do if placed near its ſurface; but ſo ſmall a force could produce no ſenſible effect. If two ſuch ſpheres were diſtant but by $\frac{1}{4}$ of an inch, they would not, even in ſpaces void of reſiſtance, come together by the force of their mutual attraction in leſs than a month's time; and leſs ſpheres will come together at a rate yet ſlower, viz. in the proportion of their diameters. Nay, whole mountains will not be ſufficient to produce any ſenſible effect. A mountain of an hemiſpherical figure, three miles high, and ſix broad, will not, by its attraction, draw the pendulum two minutes out of the true perpendicular; and it is only in the great bodies of the planets that theſe forces are to be perceived, unleſs we may reaſon about ſmaller bodies in manner following.



Let ABCD (p. 27, vol. 1) repreſent the globe of the earth cut by any plane AC into two parts ACB, and ACD. The part ACB bearing upon the part ACD preſſes it with its whole weight; nor can the part ACD ſuſtain this preſſure, and continue unmoved, if it is not oppoſed by an equal contrary preſſure. And therefore the parts equally preſs each other by their weights, that is, equally attract each other, according to the third law of motion; and, if ſeparated and let go, would fall towards each other with velocities reciprocally as the bodies. All which we may try and ſee in the load-ſtone, whoſe attracted part does not propel the part attracting, but is only ſtopped and ſuſtained thereby.

Suppoſe now that ACB repreſents ſome ſmall body on the earth's ſurface; then, becauſe the mutual attractions of this particle, and of the remaining part ACD of the earth towards each other, are equal, but the attraction of the particle towards the earth (or its weight) is as the matter of the particle (as we have proved by the experiment of the pendulums), the attraction of the earth towards the particle will likewiſe be as the matter of the particle; and therefore the attractive forces of all terreſtrial bodies will be as their ſeveral quantities of matter.

The forces (p. 175), which are as the matter in terreſtrial bodies of all forms, and therefore are not mutable with the forms, muſt be found in all ſorts of bodies whatſoever, celeſtial as well as terreſtrial, and be in all proportional to their quantities of matter, becauſe among all there is no difference of ſubſtance, but of modes and forms only. But in the celeſtial bodies the ſame thing is likewiſe proved thus. We have ſhewn that the action of the circum-ſolar force upon all the planets (reduced to equal diſtances) is as the matter of the planets; that the action of the circum-jovial force upon the ſatellites of Jupiter obſerves the ſame law; and the ſame thing is to be ſaid of the attraction of all the planets towards every planet: bnt thence it follows (by prop. 69) that their attractive forces are as their ſeveral quantities of matter.

As the parts of the earth mutually attract one another, ſo do thoſe of all the planets. If Jupiter and its ſatellites were brought together, and formed into one globe, without doubt they would continue mutually to attract one another as before. And, on the other hand, if the body of Jupiter was broke into more globes, to be ſure, theſe would no leſs attract one another than they do the ſatellites now. From theſe attractions it is that the bodies of the earth and all the planets effect a ſpherical figure, and their parts cohere, and are not diſperſed through the æther. But we have before proved that theſe forces ariſe from the univerſal nature of matter (p. 177), and that, therefore, the force of any whole globe is made up of the ſeveral forces of all its parts. And from thence it follows (by cor. 3, prop. 74) that the force of every particle decreaſes in the duplicate proportion of the diſtance from that particle; and (by prop. 73 and 75) that the force of an entire globe, reckoning from the ſurface outwards, decreaſes in the duplicate, but, reckoning inwards, in the ſimple proportion of the diſtances from the centres, if the matter of the globe be uniform. And though the matter of the globe, reckoning from the centre towards the ſurface, is not uniform (p. 177, 178), yet the decreaſe in the duplicate proportion of the diſtance outwards would (by prop. 76) take place, provided that difformity is ſimilar in places round about at equal diſtances from the centre. And two ſuch globes will (by the ſame propoſition) attract one the other with a force decreaſing in the duplicate proportion of the diſtance between their centres.

Wherefore the abſolute force of every globe is as the quantity of matter which the globe contains; but the motive force by which every globe is attracted towards another, and which, in terreſtrial bodies, we commonly call their weight, is as the content under the quantities of matter in both globes applied to the ſquare of the diſtance between their centres (by cor. 4, prop. 76), to which force the quantity of motion, by which each globe in a given time will be carried towards the other, is proportional. And the accelerative force, by which every globe according to its quantity of matter is attracted towards another, is as the quantity of matter in that other globe applied to the ſquare of the diſtance between the centres of the two (by cor. 2, prop. 76); to which force, the velocity by which the attracted globe will, in a given time, be carried towards the other is proportional. And from theſe principles well underſtood, it will be now eaſy to determine the motions of the celeſtial bodies among themſelves.

From comparing the forces of the planets one with another, we have above ſeen that the circum-ſolar does more than a thouſand times exceed all the reſt; but by the action of a force ſo great it is unavoidable but that all bodies within, nay, and far beyond, the bounds of the planetary ſyſtem muſt deſcend directly to the ſun, unleſs by other motions they are impelled towards other parts: nor is our earth to be excluded from the number of ſuch bodies; for certainly the moon is a body of the ſame nature with the planets, and ſubject to the ſame attractions with the other planets, ſeeing it is by the circum-terreſtrial force that it is retained in its orbit. But that the earth and moon are equally attracted towards the ſun, we have above proved; we have likewiſe before proved that all bodies are ſubject to the ſaid common laws of attraction. Nay, ſuppoſing any of thoſe bodies to be deprived of its circular motion about the ſun, by having its diſtance from the ſun, we may find (by prop. 36) in what ſpace of time it would in its deſcent arrive at the ſun; to wit, in half that periodic time in which the body might be revolved at one half of its former diſtance; or in a ſpace of time that is to the periodic time of the planet as 1 to $4\surd 2$; as that Venus in its deſcent would arrive at the ſun in the ſpace of 40 days, Jupiter in the ſpace of two years and one month, and the earth and moon together in the ſpace of 66 days and 19 hours. But, ſince no ſuch thing happens, it muſt needs be, that thoſe bodies are moved towards other parts (p. 3, vol. 1), nor is every motion ſufficient for this purpoſe. To hinder ſuch a deſcent, a due proportion of velocity is required. And hence depends the force of the argument drawn from the retardation of the motions of the planets. Unleſs the circum-ſolar force decreaſed in the duplicate ratio of their increaſing ſlowneſs, the exceſs thereof would force thoſe bodies to deſcend to the ſun; for inſtance, if the motion (*cæteris paribus*) was retarded by one half, the planet would be retained in its orb by one fourth of the former circum-ſolar force, and by the exceſs of the other three fourths would deſcend to the ſun. And therefore the planets (Saturn, Jupiter, Mars, Venus, and Mercury) are not really retarded in their perigees, nor become really ſtationary, or regreſſive with ſlow motions. All theſe are but apparent, and the abſolute motions, by which the planets continue to revolve in their orbits, are always direct, and nearly equable. But that ſuch motions are performed about the ſun, we have already proved; and therefore the ſun, as the centre of the abſolute motions, is quieſcent. For we can by no means allow quieſcence to the earth, leſt the planets in their perigees ſhould indeed be truly retarded, and become truly ſtationary and regreſſive, and ſo for want of motion ſhould deſcend to the ſun. But farther; ſince the planets (Venus, Mars, Jupiter, and the reſt) by radii drawn to the ſun deſcribe regular orbits, and areas (as we have ſhewn) nearly and to ſenſe proportional to the times, it follows (by prop. 3, and cor. 3, prop. 65) that the ſun is moved with no notable force, unleſs perhaps with ſuch as all the planets are equally moved with, according to their ſeveral quantities of matter, in parallel lines, and ſo the whole ſyſtem is transfered in right lines. Reject that tranſlation of the whole ſyſtem, and the ſun will be almoſt quieſcent in the centre thereof. If the ſun was revolved about the earth, and carried the other planets round about itſelf, the earth ought to attract the ſun with a great force, but the circum-ſolar planets with no force producing any ſenſible effect, which is contrary to prop. 3, cor. 65. Add to this, that if hitherto the earth, becauſe of the gravitation of its parts, has been placed by moſt authors in the lowermoſt region of the univerſe; now, for better reaſon, the ſun poſſeſſed of a centripetal force exceeding our terreſtrial gravitation a thouſand times and more, ought to be depreſſed into the lowermoſt place, and to be held for the centre of the ſyſtem. And thus the true diſpoſition of the whole ſyſtem will be more fully and more exactly underſtood.

Becauſe the fixed ſtars are quieſcent one in reſpect of another (p. 182, 183), we may conſider the ſun, earth, and planets, as one ſyſtem of bodies carried hither and thither by various motions among themſelves; and the commom centre of gravity of all (by cor. 4 of the laws of motion) will either be quieſcent, or move uniformly forward in a right line: in which caſe the whole ſyſtem will likewiſe move uniformly forward in right lines. But this is an hypotheſis hardly to be admitted; and, therefore, ſetting it aſide, that common centre will be quieſcent: and from it the ſun is never far removed. The common centre of gravity of the ſun and Jupiter falls on the ſurface of the ſun; and though all the planets were placed towards the ſame parts from the ſun with Jupiter the common centre of the ſun and all of them would ſcarcely recede twice as far from the ſun's centre; and, therefore, though the ſun, according to the various ſituation of the planets, is variouſly agitated, and always wandering to and fro with a ſlow motion of libration, yet it never recedes one entire diameter of its own body from the quieſcent centre of the whole ſyſtem. But from the weights of the ſun and planets above determined, and the ſituation of all among themſelves, their common centre of gravity may be found; and, this being given, the ſun's place to any ſuppoſed time may be obtained.

About the ſun thus librated the other planets are revolved in elliptic orbits (p. 184), and, by radii drawn to the ſun, deſcribe areas nearly proportional to the times, as is explained in prop. 65. If the ſun was quieſcent, and the other planets did not act mutually one upon another, their orbits would be elliptic, and the areas exactly proportional to the times (by prop. 11, and cor. 1, prop. 13). But the actions of the planets among themſelves, compared with the actions of the ſun on the planets, are of no moment, and produce no ſenſible errors. And thoſe errors are leſs in revolutions about the ſun agitated in the manner but now deſcribed than if thoſe revolutions were made about the the ſun quieſcent (by prop. 66, and cor. prop. 68), eſpecially if the focus of every orbit is placed in the common centre of gravity of all the lower included planets; viz. the focus of the orbit of Mercury in the centre of the ſun; the focus of the orbit of Venus in the common centre of gravity of Mercury and the ſun; the focus of the orbit of the earth in the common centre of gravity of Venus, Mercury, and the ſun; and ſo of the reſt. And by this means the foci of the orbits of all the planets, except Saturn, will not be ſenſibly removed from the centre of the ſun, nor will the focus of the orbit of Saturn recede ſenſibly from the common centre of gravity of Jupiter and the ſun. And therefore aſtronomers are not far from the truth, when they reckon the ſun's centre the common focus of all the planetary orbits. In Saturn itſelf the error thence ariſing does not exceed 1′ 45″. And if its orbit, by placing the focus thereof in the common centre of gravity of Jupiter and the ſun, ſhall happen to agree better with the phænomena, from thence all that we have ſaid will be farther confirmed.

If the ſun was quieſcent, and the planets did not act one on another, the aphelions and nodes of their orbits would likewiſe (by prop. 1, 11, and cor. prop. 13) be quieſcent. And the longer axes of their elliptic orbits would (by prop. 15) be as the cubic roots of the ſquares of their periodic times: and therefore from the given periodic times would be alſo given. But thoſe times are to be meaſured not from the equinoctial points, which are moveable, but from the firſt ſtar of Aries. Put the ſemi-axis of the earth's orbit 100000, and the ſemi-axes of the orbits of Saturn, Jupiter, Mars, Venus, and Mercury, from their periodic times, will come out 953806, 520116, 152399, 72333, 38710 reſpectively. But from the ſun's motion every ſemi-axis is increaſed (by prop. 60) by about one third of the diſtance of the ſun's centre from the common centre of gravity of the ſun and planet (p. 185, 186.) And from the actions of the exterior planets on the interior, the periodic times of the interior are ſomething protracted, though ſcarcely by any ſenſible quantity; and their aphelions are transfered (by cor. 6 and 7, prop. 66) by very ſlow motions in *conſequentia*. And on the like account the periodic times of all, eſpecially of the exterior planets, will be prolonged by the actions of the comets, if any ſuch there are, without the orb of Saturn, and the aphelions of all will be thereby carried forwards in *conſequentia*. But from the progreſs of the aphelions the regreſs of the nodes follows (by cor. 11, 13, prop. 66). And if the plane of the ecliptic is quieſcent, the regreſs of the nodes (by cor. 16, prop. 66) will be to the progreſs of the aphelion in every orbit as the regreſs of the nodes of the moon's orbit to the progreſs of its apogeon nearly, that is, as about 10 to21. But aſtronomical obſervations ſeem to confirm a very ſlow progreſs of the aphelions, and a regreſs of the nodes in reſpect of the fixed ſtars. And hence it is probable that there are comets in the regions beyond the planets, which, revolving in very eccentric orbs, quickly fly through their perihelion parts, and by an exceedingly ſlow motion in their aphelions ſpend almoſt their whole time in the regions beyond the planets; as we ſhall afterwards explain more at large.

The planets thus revolved about the ſun (p. 197, 198, 199) may at the ſame time carry others revolving about themſelves as ſatellites or moons, as appears by prop. 66. But from the action of the ſun our moon muſt move with greater velocity, and, by a radius drawn to the earth, deſcribe an area greater for the time; it muſt have its orbit leſs curve, and therefore approach nearer to the earth in the ſyzygies than in the quadratures, except in ſo far as the motion of eccentricity hinders thoſe effects. For the eccentricity is greateſt when the moon's apogeon is in the ſyzygies, and leaſt when the ſame is in the quadratures; and hence it is that the perigeon moon is ſwifter and nearer to us, but the apogeon moon ſlower and farther from us, in the ſyzygies than in the quadratures. But farther; the apogeon has a progreſſive and the nodes a regreſſive motion, both unequable. For the apogeon is more ſwiftly progreſſive in its ſyzygies, more ſlowly regreſſive in its quadratures, and by the exceſs of its progreſs above its regreſs is yearly transfered in *conſequentia*: but the nodes are quieſcent in their ſyzygies, and moſt ſwiftly regreſſive in their quadratures. But farther, ſtill, the greateſt latitude of the moon is greater it its quadratures than in its ſyzygies; and the mean motion ſwifter in the aphelion of the earth than in its perihelion. More inequalities in the moon's motion have not hitherto been taken notice of by aſtronomers: but all theſe follow from our principles in cor. 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, prop. 66, and are known really to exiſt in the heavens. And this may be ſeen in that moſt ingenious, and, if I miſtake not of all, the moſt accurate, hypotheſis of Mr *Horrox,* which Mr. *Flamſted* has fitted to the heavens; but the aſtronomical hypotheſes are to be corrected in the motion of the nodes; for the nodes admit the greateſt equation or proſthaphæreſis in their octants, and this inequality is moſt conſpicuous when the moon is in the nodes, and therefore alſo in the octants; and hence it was that *Tycho,* and others after him, referred this inequality to the octants of the moon, and made it menſtrual; but the reaſons by us adduced prove that it ought to be referred to the octants of the nodes, and to be made annual.

Beſide thoſe inequalities taken notice of by aſtronomers (p. 198, 235 to 237), there are yet ſome others, by which the moon's motions are ſo diſturbed, that hitherto by no law could they be reduced to any certain regulation. For the velocities or horary motions of the apogee and nodes of the moon, and their equations, as well as the difference betwixt the greateſt eccentricity in the ſyzygies and the leaſt in the quadratures, and that inequality which we call the variation, in the progreſs of the year are augmented and diminiſhed (by cor. 14, prop. 66) in the triplicate ratio of the ſun's apparent diameter. Beſide that, the variation is mutable nearly in the duplicate ratio of the time between the quadratures (by cor. 1 and 2, lem. 10, and cor. 16, prop. 66); and all thoſe inequalities are ſomething greater in that part of the orbit which reſpects the ſun than in the oppoſite part, but by a difference that is ſcarcely or not at all perceptible.

By a computation (p. 209), which for brevity's ſake I do not deſcribe, I alſo find that the area which the moon by a radius drawn to the earth deſcribes in the ſeveral equal moments of time is nearly as the ſum of the number $237\frac{3}{10}$, and verſed ſine of the double diſtance of the moon from the neareſt quadrature in a circle whoſe radius is unity; and therefore that the ſquare of the moon's diſtance from the earth is as that ſum divided by the horary motion of the moon. Thus it is when the variation in the octants is in its mean quantity; but if the variation is greater or leſs, that verſed ſine muſt be augmented or diminiſhed in the ſame ratio. Let aſtronomers try how exactly the diſtances thus found will agree with the moon's apparent diameters.

From the motions of our moon we may derive the motions of the moons or ſatellites of Jupiter and Saturn (p. 199); for the mean motion of the nodes of the outmoſt ſatellite of Jupiter is to the mean motion of the nodes of our moon in a proportion compounded of the duplicate proportion of the periodic time of the earth about the ſun to the periodic time of Jupiter about the ſun, and the ſimple proportion of the periodic time of the ſatellite about Jupiter to the periodic time of our moon about the earth (by cor. 16, prop. 66): and therefore thoſe nodes, in the ſpace of a hundred years, are carried 8° 24′ backwards, or in *antecendentia*. The mean motions of the nodes of the inner ſatellites are to the (mean) motion of (the nodes of) the outmoſt as their periodic times to the periodic time of this, by the ſame corollary, and are thence given. And the motion of the apſis of every ſatellite in *conſequentia* is to the motion of its nodes in *antecedentia,* as the motion of the apogee of our moon to the motion of its nodes (by the ſame corollary), and is thence given. The greateſt equations of the nodes and line of the apſes of each ſatellite are to the greateſt equations of the nodes and the line of the apſes of the moon reſpectively as the motion of the nodes and line of the apſes of the ſatellites in the time of one revolution of the firſt equations to the motion of the nodes and apogeon of the moon in the time of one revolution of the laſt equations. The variation of a ſatellite ſeen from Jupiter is to the variation of our moon in the ſame proportion as the whole motions of their nodes reſpectively, during the times in which the ſatellite and our moon (after parting from) are revolved (again) to the ſun, by the ſame corollary; and therefore in the outmoſt ſatellite the variation does not exceed 5″ 12‴. From the ſmall quantity of thoſe inequalities, and the ſlowneſs of the motions, it happens that the motions of the ſatellites are found to be ſo regular, that the more modern aſtronomers either deny all motion to the nodes, or affirm them to be very ſlowly regreſſive.

(P. 186). While the planets are thus revolved in orbits about remote centres, in the mean time they make their ſeveral rotations about their proper axes; the ſun in 26 days; Jupiter in 9^h. 56′; Mars in $24\frac{2}{3}^h$.; Venus in 23^h.; and that in planes not much inclined to the plane of the ecliptic, and according to the order of the ſigns, as aſtronomers determine from the ſpots or maculæ that by turns preſent themſelves to our ſight in their bodies; and there is a like revolution of our earth performed in 24^h.; and thoſe motions are neither accelerated nor retarded by the actions of the centripetal forces, as appears by cor. 22, prop. 66; and therefore of all others they are the moſt equable and moſt fit for the menſuration of time; but thoſe revolutions are to be reckoned equable not from their return to the ſun, but to ſome fixed ſtar: for as the poſition of the planets to the ſun is unequably varied, the revolutions of thoſe planets from ſun to ſun are rendered unequable.

In like manner is the moon revolved about its axis by a motion moſt equable in reſpect of the fixed ſtars, viz. in 27^d. 7^h. 43′, that is, in the ſpace of a ſidereal month; ſo that this diurnal motion is equal to the mean motion of the moon in its orbit: upon which account, the ſame face of the moon always reſpects the centre about which this mean motion is performed, that is, the exterior focus of the moon's orbit nearly; and hence ariſes a deflection of the moon's face from the earth, ſometimes towards the eaſt, and other times towards the weſt, according to the poſition of the focus which it reſpects; and this deflection is equal to the equation of the moon's orbit, or to the difference betwixt its mean and true motions; and this is the moon's libration in longitude: but it is likewiſe affected with a libration in latitude ariſing from the inclination of the moon's axis to the plane of the orbit in which the moon is revolved about the earth; for that axis retains the ſame poſition to the fixed ſtars nearly, and hence the poles preſent themſelves to our view by turns, as we may underſtand from the example of the motion of the earth, whoſe poles, by reaſon of the inclination of its axis to the plane of the ecliptic, are by turns illuminated by the ſun. To determine exactly the poſition of the moon's axis to the fixed ſtars, and the variation of this poſition, is a problem worthy of an aſtronomer.

By reaſon of the diurnal revolutions of the planets, the matter which they contain endeavours to recede from the axis of this motion; and hence the fluid parts riſing higher towards the equator than about the poles (p. 187), would lay the ſolid parts about the equator under water, if thoſe parts did not riſe alſo (p. 187, 191, 192): upon which account the planets are ſomething thicker about the equator than about the poles; and their equinoctial points (p. 197) thence become regreſſive; and their axes, by a motion of nutation, twice in every revolution, librate towards their ecliptics, and twice return again to their former inclination, as is explained in cor. 18, prop. 66; and hence it is that Jupiter, viewed through very long teleſcopes, does not appear altogether round (p. 191), but having its diameter that lies parallel to the ecliptic ſomething longer than that which is drawn from north to ſouth.

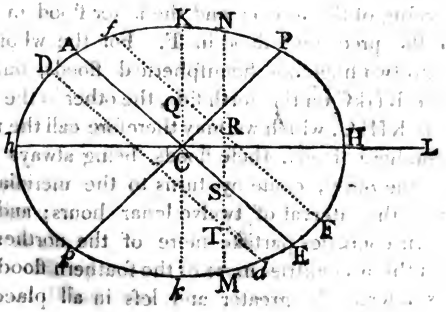
And from the diurnal motion and the attractions (p. 200, 204) of the ſun and moon our ſea ought twice to riſe and twice to fall every day, as well lunar as ſolar (by cor. 19, 20, prop. 66), and the greateſt height of the water to happen before the ſixth hour of either day and after the twelfth hour preceding. By the ſlowneſs of the diurnal motion the flood is retracted to the twelfth hour; and by the force of the motion of reciprocation it is protracted and deferred till a time nearer to the ſixth hour. But till that time is more certainly determined by the phænomena, chooſing the middle between thoſe extremes, why may we not conjecture the greateſt height of the water to happen at the third hour? for thus the water will riſe all that time in which the force of the luminaries to raiſe it is greater, and will fall all that time in which their force is leſs; viz. from the ninth to the third hour when that force is greater, and from the third to the ninth when it is leſs. The hours I reckon from the appulſe of each luminary to the meridian of the place, as well under as above the horizon; and by the hours of the lunar day I underſtand the twenty-fourth parts of that time which the moon ſpends before it comes about again by its apparent diurnal motion to the meridian of the place which it left the day before.

But the two motions which the two luminaries raiſe will not appear diſtinguiſhed, but will make a certain mixed motion. In the conjunction or oppoſition of the luminaries their forces will be conjoined, and bring on the greateſt flood and ebb. In the quadratures the ſun will raiſe the waters which the moon depreſſeth, and depreſs the waters which the moon raiſeth; and from the difference of their forces the ſmalleſt of all tides will follow. And becauſe (as experience tells us) the force of the moon is greater than that of the ſun, the greateſt height of the water will happen about the third lunar hour. Out of the ſyzygies and quadratures the greateſt tide which by the ſingle force of the moon ought to fall out at the third lunar hour, and by the ſingle force of the ſun at the third ſolar hour, by the compounded forces of both muſt fall out in an intermediate time that approaches nearer to the third hour of the moon than to that of the ſun; and, therefore, while the moon is paſſing from the ſyzygies to the quadratures, during which time the third hour of the ſun precedes the third of the moon, the greateſt tide will precede the third lunar hour, and that by the greateſt interval a little after the octants of the moon; and by like intervals the greateſt tide will follow the third lunar hour, while the moon is paſſing from the quadratures to the ſyzygies.

But the effects of the luminaries depend upon their diſtances from the earth; for when they are leſs diſtant their effects are greater, and when more diſtant their effects are leſs, and that in the triplicate proportion of their apparent diameters. Therefore it is that the ſun in the winter time, being then in its perigee, has a greater effect, and makes the tides in the ſyzygies ſomething greater, and thoſe in the quadratures ſomething leſs, *cæteris paribus*, than in the ſummer ſeaſon;

and every month the moon, while in the perigee, raiſeth greater tides than at the diſtance of 15 days before or after, when it is in its apogee. Whence it comes to paſs that two higheſt tides do not follow one the other in two immediately ſucceeding ſyzygies.

The effect of either luminary doth likewiſe depend upon its declination or diſtance from the equator; for if the luminary was placed at the pole, it would conſtantly attract all the parts of the waters, without any intenſion or remiſſion of its action, and could cauſe no reciprocation of motion; and, therefore, as the luminaries decline from the equator towards either pole, they will by degrees loſe their force, and on this account will excite leſſer tides in the ſolſtitial than in the equinoctial ſyzygies. But in the ſolſtitial quadratures they will raiſe greater tides than in the quadratures about the equinoxes; becauſe the effect of the moon, then ſituated in the equator, moſt exceeds the effect of the ſun; therefore the greateſt tides fall out in thoſe ſyzygies, and the leaſt in thoſe quadratures, which happen about the time of both equinoxes; and the greateſt tide in the ſyzygies is always ſucceeded by the leaſt tide in the quadratures, as we find by experience. But becauſe the ſun is leſs diſtant from the earth in winter than in ſummer, it comes to paſs that the greateſt and leaſt tides more frequently appear before than after the vernal equinox, and more frequently after than before the autumnal.



Moreover, the effects of the luminaries depend upon the latitudes of places. Let ApEP repreſent the earth on all ſides covered with deep waters; C its centre; P, p, its poles; AE the equator; F any place without the equator; Ff the parallel of the place; Dd the correſpondent parallel on the other ſide of the equator; L the place which the moon poſſeſſed three hours before; H the place of the earth directly under it; h the oppoſite place; K, k, the places at 90 degrees diſtance; CH, Ch, the greateſt heights of the ſea from the centre of the earth; and CK, Ck, the leaſt heights: and if with the axes Hh, Kk, an ellipſis is deſcribed, and by the revolution of that ellipſis about its longer axis Hh a ſpheroid HPKhpk is formed, this ſpheroid will nearly repreſent the figure of the ſea; and CF, Cf, CD, Cd, will repreſent the ſea in the places F, f, D, d. But farther; if in the ſaid revolution of the ellipſis any point N deſcribes the circle NM, cutting the parallels Ff, Dd, in any places R, T, and the equator AE in S, CN will repreſent the height of the ſea in all thoſe places R, S, T, ſituated in this circle. Wherefore in the diurnal revolution of any place F the greateſt flood will be in F, at the third hour after the appulſe of the moon to the meridian above the horizon; and afterwards the greateſt ebb in Q, at the third hour after the ſetting of the moon; and then the greateſt flood in f, at the third hour after the appulſe of the moon to the meridian under the horizon; and, laſtly, the greateſt ebb in Q, at the third hour after the riſing of the moon; and the latter flood in f will be leſs than the preceding flood in F. For the whole ſea is divided into two huge and hemiſpherical floods, one in the hemiſphere KHkC on the north ſide, the other in the oppoſite hemiſphere KHkC, which we may therefore call the northern and the ſouthern floods: theſe floods, being always oppoſite the one to the other, come by turns to the meridians of all places after the interval of twelve lunar hours; and, ſeeing the northern countries partake more of the northern flood, and the ſouthern countries more of the ſouthern flood, thence ariſe tides alternately greater and leſs in all places without the equator in which the luminaries riſe and ſet. But the greater tide will happen when the moon declines towards the vertex of the place, about the third hour after the appulſe of the moon to the meridian above the horizon; and when the moon changes its declination, that which was the greater tide will be changed into a leſſer; and the greateſt difference of the floods will fall out about the times of the ſolſtices, eſpecially if the aſcending node of the moon is about the firſt of Aries. So the morning tides in winter exceed thoſe of the evening, and the evening tides exceed thoſe of the morning in ſummer; at *Plymouth* by the height of one foot, but at *Briſtol* by the height of 15 inches, according to the obſervations of *Colepreſs* and *Sturmy*.

But the motions which we have been deſcribing ſuffer ſome alteration from that force of reciprocation which the waters [having once received] retain a little while by their *vis inſita*; whence it comes to paſs that the tides may continue for ſome time, though the actions of the luminaries ſhould ceaſe. This power of retaining the impreſſed motion leſſens the difference of the alternate tides, and makes thoſe tides which immediately ſucceed after the ſyzygies greater, and thoſe which follow next after the quadratures leſs. And hence it is that the alternate tides at *Plymouth* and *Briſtol* do not differ much more one from the other than by the height of a foot, or of 15 inches; and that the greateſt tides of all at thoſe ports are not the firſt but the third after the ſyzygies.

And, beſides, all the motions are retarded in their paſſage through ſhallow channels, ſo that the greateſt tides of all, in ſome ſtraits and mouths of rivers, are the fourth, or even the fifth, after the ſyzygies.

It may alſo happen that the greateſt tide may be the fourth or fifth after the ſyzygies, or fall out yet later, becauſe the motions of the ſea are retarded in paſſing through ſhallow places towards the ſhores; for ſo the tide arrives at the weſtern coaſt of *Ireland* at the third lunar hour, and an hour or two after at the ports in the ſouthern coaſt of the ſame iſland; as alſo at the iſlands *Caſſiterides,* commonly *Sorlings;* then ſucceſſively at *Falmouth, Plymouth, Portland,* the iſle of *Wight, Wincheſter, Dover,* the mouth of the *Thames,* and *London Bridge,* ſpending twelve hours in this paſſage. But farther; the propagation of the tides may be obſtructed even by the channels of the ocean itſelf, when they are not of depth enough; for the flood happens at the third lunar hour in the *Canary* iſlands; and at all thoſe weſtern coaſts that lie towards the Atlantic ocean, as of *Ireland, France, Spain*, and all *Africa*, to the *Cape* of *Good Hope*, except in ſome ſhallow places, where it is impeded, and falls out later; and in the ſtraits of *Gibraltar*, where, by reaſon of a motion propagated from the mediterranean ſea, it flows ſooner. But, paſſing from thoſe coaſts over the breadth of the ocean to the coaſts of *America*, the flood arrives firſt at the moſt eaſtern ſhores of *Braſil*, about the fourth or fifth lunar hour; then at the mouth of the river of the *Amazons* at the fixth hour, but at the neighbouring iſlands at the fourth hour; afterwards at the iſlands of *Bermudas* at the ſeventh hour, and at port St. *Auguſtin* in *Florida* at ſeven and a half. And therefore the tide is propagated through the ocean with a ſlower motion than it ſhould be according to the courſe of the moon; and this retardation is very neceſſary, that the ſea at the ſame time may fall between *Braſil* and *New France*, and riſe at the *Canary* iſlands, and on the coaſts of *Europe* and *Africa*, and *vice verſa*: for the ſea cannot riſe in one place but by falling in another. And it is probable that the *Pacific* ſea is agitated by the ſame laws; for in the coaſts of *Chili* and *Peru* the higheſt flood is ſaid to happen at the third lunar hour. But with what velocity it is thence propagated to the eaſtern coaſts of *Japan*, the *Philippine* and other iſlands adjacent to *China*, I have not yet learned.

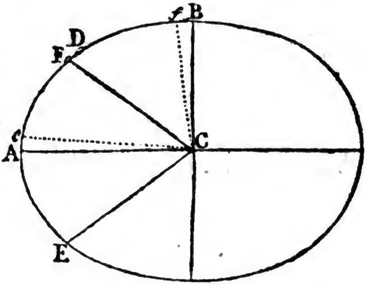
Farther; it may happen (p. 204) that the tide may be propagated from the ocean through different channels towards the ſame port, and may paſs quicker through ſome channels than through others, in which caſe the ſame tide, divided into two or more ſucceeding one another, may compound new motions of different kinds. Let us ſuppoſe one tide to be divided into two equal tides, the former whereof precedes the other by the ſpace of ſix hours, and happens at the third or twenty-ſeventh hour from the appulſe of the moon to the meridian of the port. If the moon at the time of this appulſe to the meridian was in the equator, every ſix hours alternately there would ariſe equal floods, which, meeting with as many equal ebbs, would ſo balance one the other, that, for that day, the water would ſtagnate, and remain quiet. If the moon then declined from the equator, the tides in the ocean would be alternately greater and leſs, as was ſaid; and from hence two greater and two leſſer tides would be alternately propagated towards that port. But the two greater floods would make the greateſt height of the waters to fall out in the middle time betwixt both, and the greater and leſſer floods would make the waters to riſe to a mean height in the middle time between them; and in the middle time between the two leſſer floods the waters would riſe to their leaſt height. Thus in the ſpace of twenty hours the waters would come, not twice, but once only to their greateſt, and once only to their leaſt height; and their greateſt height, if the moon declined towards the elevated pole, would happen at the ſixth or thirtieth hour after the appulſe of the moon to the meridian; and when the moon changed its declination, this flood would be changed into an ebb.

Of all which we have an example in the port of *Batſham*, in the kingdom of *Tunquin*, in the latitude of 20° 50′ north. In that port, on the day which follows after the paſſage of the moon over the equator, the waters ſtagnate; when the moon declines to the north, they begin to flow and ebb, not twice, as in other ports, but once only every day; and the flood happens at the ſetting, and the greateſt ebb at the riſing of the moon. This tide increaſeth with the declination of the moon till the ſeventh or eighth day; then for the ſeventh or eighth day following it decreaſeth at the ſame rate as it had increaſed before, and ceaſeth when the moon changeth its declination. After which the flood is immediately changed into an ebb; and thenceforth the ebb happens at the ſetting and the flood at the riſing of the moon, till the moon again changes its declination. There are two inlets from the ocean to this port; one more direct and ſhort between the iſland *Hainan* and the coaſt of *Quantung*, a province of *China*; the other round about between the ſame iſland and the coaſt of

Cochim; and through the ſhorter paſſage the tide is ſooner propagated to *Batſham*.

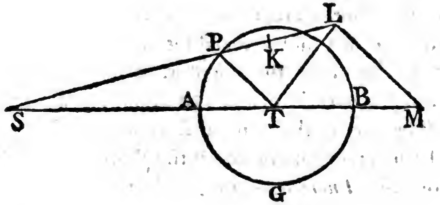
In the channels of rivers the influx and reflux depends upon the current of the rivers, which obſtructs the ingreſs of the waters from the ſea, and promotes their egreſs to the ſea, making the ingreſs later and ſlower, and the egreſs ſooner and faſter; and hence it is that the reflux is of longer duration than the influx, eſpecially far up the rivers, where the force of the ſea is leſs. So *Sturmy* tells us, that in the river *Avon*, three miles below *Briſtol*, the water flows only five hours, but ebbs ſeven; and without doubt the difference is yet greater above *Briſtol*, as at *Careſham* or the *Bath*. This difference does likewiſe depend upon the quantity of the flux and reflux; for the more vehement motion of the ſea near the ſyzygies of the luminaries more eaſily overcoming the reſiſtance of the rivers, will make the ingreſs of the water to happen ſooner and to continue longer, and will therefore diminiſh this difference. But while the moon is approaching to the ſyzygies, the rivers will be more plentifully filled, their currents being obſtructed by the greatneſs of the tides, and therefore will ſomething more retard the reflux of the ſea a little after than a little before the ſyzygies. Upon which account the ſloweſt tides of all will not happen in the ſyzygies, but prevent them a little; and I obſerved above that the tides before the ſyzygies were alſo retarded by the force of the ſun; and from both cauſes conjoined the retardation of the tides will be both greater and ſooner before the ſyzygies. All which I find to be ſo, by the tide-tables which *Flamſted* has compoſed from a great many obſervations.

By the laws we have been deſcribing, the times of the tides are governed; but the greatneſs of the tides depends upon the greatneſs of the ſeas. Let C repreſent the centre of the earth, EADB the oval figure of the ſeas, CA the longer ſemi-axis of this oval, CB the ſhorter inſiſting at right angles upon the former, D the middle point between A and B, and ECF or eCf the angle at the centre of the earth, ſubtended by the breadth of the ſea that terminates in the ſhores E, F, or e, f. Now, ſuppoſing that the point A is in the mid-



dle between the points E, F, and the point D in the middle between the points e, f, if the difference of the heights CA, CB, repreſent the quantity of the tide in a very deep ſea ſurrounding the whole earth, the exceſs of the height CA above the height CE or CF will repreſent the quantity of the tide in the middle of the ſea EF, terminated by the ſhores E, F; and the exceſs of the height Ce above the height Cf will nearly repreſent the quantity of the tide on the ſhores f of the ſame ſea. Whence it appears that the tides are far leſs in the middle of the ſea than at the ſhores; and that the tides at the ſhores are nearly as EF (p. 243, 244), the breadth of the ſea not exceeding a quadrantal arc. And hence it is that near the equator, where the ſea between *Africa* and *America* is narrow, the tides are far leſs than towards either ſide in the temperate zones, where the ſeas are extended wider; or on almoſt all the ſhores of the *Pacific* ſea, as well towards *America* as towards *China*, and within as well as without the tropics; and that in iſlands in the middle of the ſea they ſcarcely riſe higher than two or three feet, but on the ſhores of great continents are three or four times greater, and above, eſpecially if the motions propagated from the ocean are by degrees contracted into a narrow ſpace, and the water, to fill and empty the bays alternately, is forced to flow and ebb with great violence through ſhallow places; as *Plymouth* and *Chepſtow Bridge* in *England*, at the mount of *St. Michael* and town of *Avranches* in *Normandy*, and at *Cambaia* and *Pegu* in the *Eaſt Indies*. In which places, the ſea, hurried in and out with great violence, ſometimes lays the ſhores under water, ſometimes leaves them dry, for many miles. Nor is the force of the influx and efflux to be broke till it has raiſed or depreſſed the water to forty or fifty feet and more. Thus alſo long and ſhallow ſtraits that open to the ſea with mouths wider and deeper than the reſt of their channel (ſuch as thoſe about *Britain*, and the *Magellanic Straits* at the eaſtern entry) will have a greater flood and ebb, or will more intend and remit their courſe, and therefore will riſe higher and be depreſſed lower. On the coaſt of *South America* it is ſaid that the *Pacific* ſea in its reflux ſometimes retreats two miles, and gets out of ſight of thoſe that ſtand on ſhore. Whence in theſe places the floods will be alſo higher; but in deeper waters the velocity of influx and efflux is always leſs, and therefore the aſcent and deſcent is ſo too. Nor in ſuch places is the ocean known to aſcend to more than ſix, eight, or ten feet. The quantity of the aſcent I compute in the following manner.

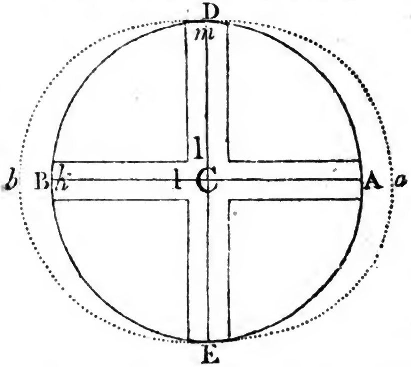
Let S repreſent the ſun, T the earth (p. 205, 206), P the moon, PAGB the moon's orbit. In SP take SK equal to ST



and SL to SK in the duplicate ratio of SK to SP. Parallel to PT draw LM; and, ſuppoſing the mean quantity of the circum-ſolar force directed towards the earth to be repreſented by the diſtance ST or SK, SL will repreſent the quantity thereof directed towards the moon. But that force is compounded of the parts SM, LM; of which the force LM, and that part of SM which is repreſented by TM, do diſturb the motion of the moon (as appears from prop. 66, and its corollaries). In ſo far as the earth and moon are revolved about their common centre of gravity, the earth will be liable to the action of the like forces. But we may refer the ſums as well of the forces as of the motions to the moon, and repreſent the ſums of the forces by the lines TM and ML, which are proportional to them. The force LM, in its mean quantity, is to the force by which the moon may be revolved in an orbit, about the earth quieſcent, at the diſtance PT in the duplicate ratio of the moon's periodic time about the earth to the earth's periodic time about the ſun (by cor. 17, prop. 66); that is, in the duplicate ratio of $27^d. 7^h. 43'$ to $365^d. 6^h. 9'$, or as 1000 to 178725, or 1 to 178$\frac{29}{40}$. The force by which the moon may be revolved in its orb about the earth in reſt, at the diſtance PT of 60½ ſemi-diameters of the earth, is to the force by which it may revolve in the ſame time at the diſtance of 60 ſemi-diameters as 60½ to 60; and this force is to the force of gravity with us as 1 to 60 × 60 nearly; and therefore the mean force ML is to the force of gravity at the ſurface of the earth as 1 × 60½ to 60 × 60 × 178$\frac{29}{40}$, or 1 to 638092,6. Whence the force TM will be alſo given from the proportion of the lines TM, ML. And theſe are the forces of the ſun, by which the moon's motions are diſturbed.

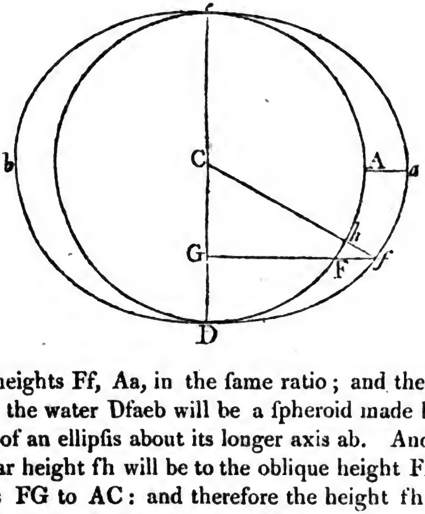
If from the moon's orbit (p. 240) we deſcend to the earth's ſurface, thoſe forces will be diminiſhed in the ratio of the diſtances 60½ and 1; and therefore the force LM will then become 38604600 times leſs than the force of gravity. But this force acting equally every where upon the earth, will ſcarcely effect any change on the motion of the ſea, and therefore may be neglected in the explication of that motion. The other force TM, in places where the ſun is vertical, or in their nadir, is triple the quantity of the force ML, and therefore but 12868200 times leſs than the force of gravity.

Suppoſe now ADBE to repreſent the ſpherical ſurface of the earth, aDbE the ſurface of the water overſpreading it, C the centre of both, A the place to which the ſun is vertical, B the place oppoſite; D, E, places at 90 degrees diſtance from the former; ACEmlk a right angled cylindric canal paſſing through the earth's centre. The force TM in any place is as the diſtance of the place from the plane DE, on which



a line from A to C inſiſts at right angles, and therefore in the part of the canal which is repreſented by EClm is of no quantity, but in the other part AClk is as the gravity at the ſeveral heights; for in deſcending towards the centre of the earth, gravity is (by prop. 73) every where as the height; and therefore the force TM drawing the water upwards will diminiſh its gravity in the leg AClk of the canal in a given ratio: upon which account the water will aſcend in this leg, till its defect of gravity is ſupplied by its greater height; nor will it reſt in an equilibrium till its total gravity becomes equal to the total gravity in EClm, the other leg of the canal. Becauſe the gravity of every particle is as its diſtance from the earth's centre, the weight of the whole water in either leg will increaſe in the duplicate ratio of the height; and therefore the height of the water in the leg AClk will be to the height thereof in the leg ClmE in the ſubduplicate ratio of the number 12868201 to 12868200, or in the ratio of the number 25623053 to the number 25623052, and the height of the water in the leg EClm to the difference of the heights, as 25623052 to 1. But the height in the leg EClm is of 19615800 *Paris* feet, as has been lately found by the menſuration of the *French*; and, therefore, by the preceding analogy, the difference of the heights comes out 9$\frac{1}{5}$ inches of the *Paris* foot; and the ſun's force will make the height of the ſea at A to exceed the height of the ſame at E by 9 inches. And though the water of the canal ACEmlk be ſuppoſed to be frozen into a hard and ſolid conſiſtence, yet the heights thereof at A and E, and all other intermediate places, would ſtill remain the ſame.

Let Aa (in the following figure) repreſent that exceſs of height of 9 inches at A, and hf the exceſs of height at any other place h; and upon DC let fall the perpendicular fG, meeting the globe of the earth in F; and becauſe the diſtance of the ſun is ſo great that all the right lines drawn thereto may be conſidered as parallel, the force TM in any place f will be to the ſame force in the place A as the ſine FG to the radius AC. And, therefore, ſince thoſe forces tend to the ſun in the direction of parallel lines, they will generate the parallel heights Ff, Aa, in the ſame ratio; and therefore the figure of the water Dfaeb will be a ſpheroid made by the revolution of an ellipſis about its longer axis ab. And the perpendicular height fh will be to the oblique height Ff as fG to fC, or as FG to AC: and therefore the height fh is to the height Aa in the duplicate ratio of FG to AC, that is, in the ratio of the verſed ſine of double the angle DCf to double the radius, and is thence given. And hence to the ſeveral moments of the apparent revolution of the ſun about the earth we may infer the proportion of the aſcent and deſcent of the waters at any given place under the equator, as well as of the diminution of that aſcent and deſcent, whether ariſing



from the latitude of places or from the ſun's declination; viz. that on account of the latitude of places, the aſcent and deſcent of the ſea is in all places diminiſhed in the duplicate ratio of the co-ſines of latitude; and on account of the ſun's declination, the aſcent and deſcent under the equator is diminiſhed in the duplicate ratio of the co-ſine of declination. And in places without the equator the half ſum of the morning and evening aſcents (that is, the mean aſcent) is diminiſhed nearly in the ſame ratio.

Let S and L reſpectively repreſent the forces of the ſun and moon placed in the equator, and at their mean diſtances from the earth; R the radius; T and V the verſed ſines of double the complements of the ſun and moon's declinations to any given time; D and E the mean apparent diameters of the ſun and moon: and, ſuppoſing F and G to be their apparent diameters to that given time, their forces to raiſe the tides under the equator will be, in the ſyzygies, $\frac{VG^3}{2RE^3}L + \frac{TF^3}{2RD^3}S$; in the quadratures, $\frac{VG^3}{2RE^3}L - \frac{TF^3}{2RD^3}S$. And if the ſame ratio is likewiſe obſerved under the parallels, from obſervations accurately made in our northern climates we may determine the proportion of the forces L and S; and then by means of this rule predict the quantities of the tides to every ſyzygy and quadrature.

At the mouth of the river *Avon*, three miles below *Briſtol* (p. 241, 242, 243, 244), in ſpring and autumn, the whole aſcent of the water in the conjunction or oppoſition of the luminaries (by the obſervation of *Sturmy*) is about 45 feet, but in the quadratures only 25. Becauſe the apparent diameters of the luminaries are not here determined, let us aſſume them in their mean quantities, as well as the moon's declination in the equinoctial quadratures in its mean quantity, that is, $23\frac{1}{2}°$; and the verſed ſine of double its complement will be 1682, ſuppoſing the radius to be 1000. But the declinations of the ſun in the equinoxes and of the moon in the ſyzygies are of no quantity, and the verſed ſines of double the complements are each 2000. Whence thoſe forces become L + S in the ſyzygies, and $\frac{1682}{2000}\mathrm{L} - \mathrm{S}$ in the quadratures, reſpectively proportional to the heights of the tides of 45 and 25 feet, or of 9 and 5 paces. And, therefore, multiplying the extremes and the means, we have $5\mathrm{L} + 5\mathrm{S} = \frac{15138}{2000}\mathrm{L} - 9\mathrm{S}$, or $\mathrm{L} = \frac{28000}{5138}\mathrm{S} = 5\frac{5}{11}\mathrm{S}$.

But farther; I remember to have been told, that in ſummer the aſcent of the ſea in the ſyzygies is to the aſcent thereof in the quadratures as about 5 to 4. In the ſolſtices themſelves it is probable that the proportion may be ſomething leſs, as about 6 to 5; whence it would follow that L is $= 5\frac{1}{6}\mathrm{S}$ [for then the proportion is $\frac{1682}{2000}\mathrm{L} + \frac{1682}{2000}\mathrm{S} : \mathrm{L} - \frac{1682}{2000}\mathrm{S} :: 6 : 5$]. Till we can more certainly determine the proportion from obſervation, let us aſſume $\mathrm{L} = 5\frac{1}{3}\mathrm{S}$; and ſince the heights of the tides are as the forces which excite them, and the force of the ſun is able to raiſe the tides to the height of nine inches, the moon's force will be ſufficient to raiſe the ſame to the height of four feet. And if we allow that this height may be doubled, or perhaps tripled, by that force of reciprocation which we obſerve in the motion of the waters, and by which their motion once begun is kept up for ſome time, there will be force enough to generate all that quantity of tides which we really find in the ocean.

Thus we have ſeen that theſe forces are ſufficient to move the ſea. But, ſo far as I can obſerve, they will not be able to produce any other effect ſenſible on our earth; for ſince the weight of one grain in 4000 is not ſenſible in the niceſt balance; and the ſun's force to move the tides is 12868200 leſs than the force of gravity; and the ſum of the forces of both moon and ſun, exceeding the ſun's force only in the ratio of $6\frac{1}{3}$ to 1, is ſtill 2032890 times leſs than the force of gravity; it is evident that both forces together are 500 times leſs than what is required ſenſibly to increaſe or diminiſh the weight of any body in a balance. And, therefore, they will not ſenſibly move any ſuſpended body; nor will they produce any ſenſible effect on pendulums, barometers, bodies ſwimming in ſtagnant water, or in the like ſtatical experiments. In the atmoſphere, indeed, they will excite ſuch a flux and reflux as they do in the ſea, but with ſo ſmall a motion that no ſenſible wind will be thence produced.

If the effects of both moon and ſun in raiſing the tides (p. 245), as well as their apparent diameters, were equal among themſelves, their abſolute forces would (by cor. 14, prop. 66) be as their magnitudes. But the effect of the moon is to the effect of the ſun as about $5\frac{1}{3}$ to 1; and the moon's diameter leſs than the ſun's in the ratio of $31\frac{1}{2}$ to $32\frac{1}{5}$, or of 45 to 46. Now the force of the moon is to be increaſed in the ratio of the effect directly, and in the triplicate ratio of the diameter inverſely. Whence the force of the moon compared with its magnitude will be to the force of the ſun compared with its magnitude in the ratio compounded of $5\frac{1}{3}$ to 1, and the triplicate of 45 to 46 inverſely, that is, in the ratio of about $5\frac{7}{10}$ to 1. And therefore the moon, in reſpect of the magnitude of its body, has an abſolute centripetal force greater than the ſun in reſpect of the magnitude of its body in the ratio of $5\frac{7}{10}$ to 1, and is therefore more denſe in the ſame ratio.

In the time of $27^d. 7^h. 43'$, in which the moon makes its revolution about the earth, a planet may be revolved about the ſun at the diſtance of 18,954 diameters of the ſun from the ſun's centre, ſuppoſing the mean apparent diameter of the ſun to be $32\frac{1}{5}'$; and in the ſame time the moon may be revolved about the earth at reſt, at the diſtance of 30 of the earth's diameters. If in both caſes the number of diameters was the ſame, the abſolute circum-terreſtrial force would (by cor. 2, prop. 72) be to the abſolute circum-ſolar force as the magnitude of the earth to the magnitude of the ſun. Becauſe the number of the earth's diameters is greater in the ratio of 30 to 18,954, the body of the earth will be leſs in the triplicate of that ratio, that is, in the ratio of $3\frac{22}{25}$ to 1. Wherefore the earth's force, for the magnitude of its body, is to the ſun's force, for the magnitude of its body, as $3\frac{22}{25}$ to 1; and conſequently the earth's denſity to the ſun's will be in the ſame ratio. Since, then, the moon's denſity is to the ſun's denſity as $5\frac{7}{10}$ to 1, the moon's denſity will be to the earth's denſity as $5\frac{7}{10}$ to $3\frac{28}{29}$, or as 23 to 16. Wherefore ſince the moon's magnitude is to the earth's magnitude as about 1 to $41\frac{1}{2}$, the moon's abſolute centripetal force will be to the earth's abſolute centripetal force as about 1 to 29, and the quantity of matter in the moon to the quantity of matter in the earth in the ſame ratio. And hence the common centre of gravity of the earth and moon is more exactly determined than hitherto has been done; from the knowledge of which we may now infer the moon's diſtance from the earth with greater accuracy. But I would rather wait till the proportion of the bodies of the moon and earth one to the other is more exactly defined from the phænomena of the tides, hoping that in the mean time the circumference of the earth may be meaſured from more diſtant ſtations than any body has yet employed for this purpoſe.

Thus I have given an account of the ſyſtem of the planets. As to the fixed ſtars, the ſmallneſs of their annual parallax proves them to be removed to immenſe diſtances from the ſyſtem of the planets: that this parallax is leſs than one minute is moſt certain; and from thence it follows that the diſtance of the fixed ſtars is above 360 times greater than the diſtance of Saturn from the ſun. Such as reckon the earth one of the planets, and the ſun one of the fixed ſtars, may remove the fixed ſtars to yet greater diſtances by the following arguments: from the annual motion of the earth there would happen an apparent tranſpoſition of the fixed ſtars, one in reſpect of another, almoſt equal to their double parallax; but the greater and nearer ſtars, in reſpect of the more remote, which are only ſeen by the teleſcope, have not hitherto been obſerved to have the leaſt motion. If we ſhould ſuppoſe that motion to be but leſs than 20″, the diſtance of the nearer fixed ſtars would exceed the mean diſtance of Saturn by above 2000 times. Again; the diſk of Saturn, which is only 17″ or 18″ in diameter, receives but about $\frac{1}{2100000000}$ of the ſun's light; for ſo much leſs is that diſk than the whole ſpherical ſurface of the orb of Saturn. Now if we ſuppoſe Saturn to reflect about $\frac{1}{4}$ of this light, the whole light reflected from its illuminated hemiſphere will be about $\frac{1}{4200000000}$ of the whole light emitted from the ſun's hemiſphere; and, therefore, ſince light is rarefied in the duplicate ratio of the diſtance from the luminous body, if the ſun was $10000\sqrt{42}$ times more diſtant than Saturn, it would yet appear as lucid as Saturn now does without its ring, that is, ſomething more lucid than a fixed ſtar of the firſt magnitude. Let us, therefore, ſuppoſe that the diſtance from which the ſun would ſhine as a fixed ſtar exceeds that of Saturn by about 100,000 times, and its apparent diameter will be $7^{v}. 16^{vi}$. and its parallax ariſing from the annual motion of the earth $13''''$: and ſo great will be the diſtance, the apparent diameter, and the parallax of the fixed ſtars of the firſt magnitude, in bulk and light equal to our ſun. Some may, perhaps, imagine that a great part of the light of the fixed ſtars is intercepted and loſt in its paſſage through ſo vaſt ſpaces, and upon that account pretend to place the fixed ſtars at nearer diſtances; but at this rate the remoter ſtars could be ſcarcely ſeen. Suppoſe, for example, that $\frac{3}{4}$ of the light periſh in its paſſage from the neareſt fixed ſtars to us; then $\frac{3}{4}$ will twice periſh in its paſſage through a double ſpace, thrice through a triple, and ſo forth. And, therefore, the fixed ſtars that are at a double diſtance will be 16 times more obſcure, viz. 4 times more obſcure on account of the diminiſhed apparent diameter; and, again, 4 times more on account of the loſt light. And, by the ſame argument, the fixed ſtars at a triple diſtance will be $9 \times 4 \times 4$, or 144 times more obſcure; and thoſe at a quadruple diſtance will be $16 \times 4 \times 4 \times 4$, or 1024 times more obſcure; but ſo great a diminution of light is no ways conſiſtent with the phænomena and with that hypotheſis which places the fixed ſtars at different diſtances.

The fixed ſtars being, therefore, at ſuch vaſt diſtances from one another (p. 254, 255), can neither attract each other ſenſibly, nor be attracted by our ſun. But the comets muſt unavoidably be acted on by the circum-ſolar force; for as the comets were placed by aſtronomers above the moon, becauſe they were found to have no diurnal parallax, ſo their annual parallax is a convincing proof of their deſcending into the regions of the planets. For all the comets which move in a direct courſe, according to the order of the ſigns, about the end of their appearance become more than ordinarily ſlow, or retrograde, if the earth is between them and the ſun; and more than ordinarily ſwift if the earth is approaching to a heliocentric oppoſition with them. Whereas, on the other hand, thoſe which move againſt the order of the ſigns, towards the end of their appearance, appear ſwifter than they ought to be if the earth is between them and the ſun; and ſlower, and perhaps retrograde, if the earth is in the other ſide of its orbit. This is occaſioned by the motion of the earth in different ſituations. If the earth go the ſame way with the comet, with a ſwifter motion, the comet becomes retrograde; if with a ſlower motion, the comet becomes ſlower however; and if the earth move the contrary way, it becomes ſwifter; and by collecting the differences between the ſlower and ſwifter motions, and the ſums of the more ſwift and retrograde motions, and comparing them with the ſituation and motion of the earth from whence they ariſe, I found, by means of this parallax, that the diſtances of the comets at the time they ceaſe to be viſible to the naked eye are always leſs than the diſtance of Saturn, and generally even leſs than the diſtance of Jupiter.

The ſame thing may be collected from the curvature of the way of the comets (p. 256). Theſe bodies go on nearly in great circles while their motion continues ſwift; but about the end of their courſe, when that part of their apparent motion which ariſes from the parallax bears a greater proportion to their whole apparent motion, they commonly deviate from thoſe circles; and when the earth goes to one ſide, they deviate to the other; and this deflection, becauſe of its correſponding with the motion of the earth, muſt ariſe chiefly from the parallax; and the quantity thereof is ſo conſiderable, as, by my computation, to place the diſappearing comets a good deal lower than Jupiter. Whence it follows, that, when they approach nearer to us in their perigees and perihelions, they often deſcend below the orbits of Mars and the inferior planets.

Moreover, this nearneſs of the comets is confirmed by the annual parallax of the orbit, in ſo far as the ſame is pretty nearly collected by the ſuppoſition that the comets move uniformly in right lines. The method of collecting the diſtance of a comet according to this hypotheſis from four obſervations (firſt attempted by *Kepler*, and perfected by Dr. *Wallis* and Sir *Chriſtopher Wren*) is well known; and the comets reduced to this regularity generally paſs through the middle of the planetary region. So the comets of the years 1607 and 1618, as their motions are defined by *Kepler*, paſſed between the ſun and the earth; that of the year 1664 below the orbit of Mars; and that in 1680 below the orbit of Mercury, as its motion was defined by Sir *Chriſtopher Wren* and others. By a like rectilinear hypotheſis, *Hevelius* placed all the comets about which we have any obſervations below the orbit of Jupiter. It is a falſe notion, therefore, and contrary to aſtronomical calculation, which ſome have entertained, who, from the regular motion of the comets, either remove them into the regions of the fixed ſtars, or deny the motion of the earth; whereas their motions cannot be reduced to perfect regularity, unleſs we ſuppoſe them to paſs through the regions near the earth in motion; and theſe are the arguments drawn from the parallax, ſo far as it can be determined without an exact knowledge of the orbits and motions of the comets.

The near approach of the comets is farther confirmed from the light of their heads (p. 257 to 260); for the light of a celeſtial body, illuminated by the ſun, and receding to remote parts, is diminiſhed in the quadruplicate proportion of the diſtance; to wit, in one duplicate proportion on account of the increaſe of the diſtance from the ſun; and in another duplicate proportion on account of the decreaſe of the apparent diameter. Hence it may be inferred, that Saturn being at a double diſtance, and having its apparent diameter nearly half of that of Jupiter, muſt appear about 16 times more obſcure; and that, if its diſtance were 4 times greater, its light would be 256 times leſs; and therefore would be hardly perceivable to the naked eye. But now the comets often equal Saturn's light, without exceeding him in their apparent diameters. So the comet of the year 1668, according to Dr. *Hooke*'s obſervations, equalled in brightneſs the light of a fixed ſtar of the firſt magnitude; and its head, or the ſtar in the middle of the coma, appeared, through a teleſcope of 15 feet, as lucid as Saturn near the horizon; but the diameter of the head was only 25″; that is, almoſt the ſame with the diameter of a circle equal to Saturn and his ring. The coma or hair ſurrounding the head was about ten times as broad; namely, 4$\frac{1}{6}$ min. Again; the leaſt diameter of the hair of the comet of the year 1682, obſerved by Mr. *Flamſted* with a tube of 16 feet, and meaſured with the micrometer, was 2′ 0″; but the nucleus, or ſtar in the middle, ſcarcely poſſeſſed the tenth part of this breadth, and was therefore only 11″ or 12″ broad; but the light and clearneſs of its head exceeded that of the year 1680, and was equal to that of the ſtars of the firſt or ſecond magnitude. Moreover, the comet of the year 1665, in *April,* as *Hevelius* informs us, exceeded almoſt all the fixed ſtars in ſplendor, and even Saturn itſelf, as being of a much more vivid colour; for this comet was more lucid than that which appeared at the end of the foregoing year, and was compared to the ſtars of the firſt magnitude. The diameter of the coma was about 6′; but the nucleus, compared with the planets by means of a teleſcope, was plainly leſs than Jupiter, and was ſometimes thought leſs, ſometimes equal to the body of Saturn within the ring. To this breadth add that of the ring, and the whole face of Saturn will be twice as great as that of the comet, with a light not at all more intenſe; and therefore the comet was nearer to the ſun than Saturn. From the proportion of the nucleus to the whole head found by theſe obſervations, and from its breadth, which ſeldom exceeds 8′ or 12′, it appears that the ſtars of the comets are moſt commonly of the ſame apparent magnitude as the planets; but that their light may be compared oftentimes with that of Saturn, and ſometimes exceeds it. And hence it is certain that in their perihelia their diſtances can ſcarcely be greater than that of Saturn. At twice that diſtance, the light would be four times leſs, which beſides by its dim paleneſs would be as much inferior to the light of Saturn as the light of Saturn is to the ſplendor of Jupiter: but this difference would be eaſily obſerved. At a diſtance ten times greater, their bodies muſt be greater than that of the ſun; but their light would be 100 times fainter than that of Saturn. And at diſtances ſtill greater, their bodies would far exceed the ſun; but, being in ſuch dark regions, they muſt be no longer viſible. So impoſſible is it to place the comets in the middle regions between the ſun and fixed ſtars, accounting the ſun as one of the fixed ſtars; for certainly they would receive no more light there from the ſun than we do from the greateſt of the fixed ſtars.

So far we have gone without conſidering that obſcuration which comets ſuffer from that plenty of thick ſmoke which encompaſſeth their heads, and through which the heads always ſhew dull as through a cloud; for by how much the more a body is obſcured by this ſmoke, by ſo much the more near it muſt be allowed to come to the ſun, that it may vie with the planets in the quantity of light which it reflects; whence it is probable that the comets deſcend far below the orbit of Saturn, as we proved before from their parallax. But, above all, the thing is evinced from their tails, which muſt be owing either to the ſun's light reflected from a ſmoke ariſing from them, and diſperſing itſelf through the æther, or to the light of their own heads.

In the former caſe we muſt ſhorten the diſtance of the comets, leſt we be obliged to allow that the ſmoke ariſing from their heads is propagated through ſuch a vaſt extent of ſpace, and with ſuch a velocity of expanſion, as will ſeem altogether incredible; in the latter caſe the whole light of both head and tail muſt be aſcribed to the central nucleus. But, then, if we ſuppoſe all this light to be united and condenſed within the diſk of the nucleus, certainly the nucleus will by far exceed Jupiter itſelf in ſplendor, eſpecially when it emits a very large and lucid tail. If, therefore, under a leſs apparent diameter, it reflects more light, it muſt be much more illuminated by the ſun, and therefore much nearer to it. So the comet that appeared *Dec.* 12 and 15, O.S. *Anno* 1679, at the time it emitted a very ſhining tail, whoſe ſplendor was equal to that of many ſtars like Jupiter, if their light were dilated and ſpread through ſo great a ſpace, was, as to the magnitude of its nucleus, leſs than Jupiter (as Mr. *Flamſted* obſerved), and therefore was much nearer to the ſun: nay, it was even leſs than Mercury. For on the 17th of that month, when it was nearer to the earth, it appeared to *Caſſini* through a teleſcope of 35 feet a little leſs than the globe of Saturn. On the 8th of this month, in the morning, Dr. *Halley* ſaw the tail, appearing broad and very ſhort, and as if it roſe from the body of the ſun itſelf, at that time very near its riſing. Its form was like that of an extraordinary bright cloud; nor did it diſappear till the ſun itſelf began to be ſeen above the horizon. Its ſplendor, therefore, exceeded the light of the clouds till the ſun roſe, and far ſurpaſſed that of all the ſtars together, as yielding only to the immediate brightneſs of the ſun itſelf. Neither Mercury, nor Venus, nor the moon itſelf, are ſeen ſo near the riſing ſun. Imagine all this dilated light collected together, and to be crowded into the orbit of the comet's nucleus which was leſs than Mercury; by its ſplendor, thus increaſed, becoming ſo much more conſpicuous, it will vaſtly exceed Mercury, and therefore muſt be nearer to the ſun. On the 12th and 15th of the ſame month, this tail, extending itſelf over a much greater ſpace, appeared more rare; but its light was ſtill ſo vigorous as to become viſible when the fixed ſtars were hardly to be ſeen, and ſoon after to appear like a fiery beam ſhining in a wonderful manner. From its length, which was 40 or 50 degrees, and its breadth of 2 degrees, we may compute what the light of the whole muſt be.

This near approach of the comets to the ſun is confirmed from the ſituation they are ſeen in when their tails appear moſt reſplendent; for when the head paſſes by the ſun, and lies hid under the ſolar rays, very bright and ſhining tails, like fiery beams, are ſaid to iſſue from the horizon; but afterwards, when the head begins to appear, and is got farther from the ſun, that ſplendor always decreaſes, and turns by degrees into a paleneſs like to that of the milky way, but much more ſenſible at firſt; after that vaniſhing gradually. Such was that moſt reſplendent comet deſcribed by *Ariſtotle*, lib. 1. Meteor. 6. "The head thereof could not be ſeen, becauſe it ſet before the ſun, or at leaſt was hid under the ſun's rays; but the next day it was ſeen as well as might be; for, having left the ſun but a very little way, it ſet immediately after it; and the ſcattered light of the head obſcured by the too great ſplendour (of the tail) did not yet appear. But afterwards (ſays *Ariſtotle*), when the ſplendour of the tail was now diminiſhed, (the head of) the comet recovered its native brightneſs. And the ſplendour of its tail reached now to a third part of the heavens (that is, to 60°). It appeared in the winter ſeaſon, and, riſing to *Orion*'s girdle, there vaniſhed away." Two comets of the ſame kind are deſcribed by *Juſtin*, lib. 37, which, according to his account, "ſhined ſo bright, that the whole heaven ſeemed to be on fire; and by their greatneſs filled up a fourth part of the heavens, and by their ſplendour exceeded that of the ſun." By which laſt words a near poſition of theſe bright comets and the riſing or ſetting ſun is intimated (p. 296, 297). We may add to theſe the comet of the year 1101 or 1106, "the ſtar of which was ſmall and obſcure (like that of 1680); but the ſplendour ariſing from it extremely bright, reaching like a fiery beam to the eaſt and north," as *Hevelius* has it from *Simeon*, the monk of *Durham*. It appeared at the beginning of *February* about the evening in the ſouth-weſt. From this and from the ſituation of the tail we may infer that the head was near the ſun. *Matthew Paris* ſays, "it was about one cubit from the ſun; from the third [or rather the ſixth] to the ninth hour ſending out a long ſtream of light." The comet of 1264, in *July*, or about the ſolſtice, preceded the riſing ſun, ſending out its beams with a great light towards the weſt as far as the middle of the heavens; and at the beginning it aſcended a little above the horizon: but as the ſun went forwards it retired every day farther from the horizon, till it paſſed by the very middle of the heavens. It is ſaid to have been at the beginning large and bright, having a large coma, which decayed from day to day. It is deſcribed in *Append. Matth. Pariſ. Hiſt. Ang.* after this manner: "*An. Chriſti* 1265, there appeared a comet ſo wonderful, that none then living had ever ſeen the like; for, riſing from the eaſt with a great brightneſs, it extended itſelf with a great light as far as the middle of the hemiſphere towards the weſt." The Latin original being ſomewhat barbarous and obſcure, it is here ſubjoined. *Ab oriente enim cum magno fulgore ſurgens, uſque ad medium hemiſphærii verſus occidentem, omnia perlucide pertrahebat.*

"In the year 1401 or 1402, the ſun being got below the horizon, there appeared in the weſt a bright and ſhining comet, ſending out a tail upwards, in ſplendor like a flame of fire, and in form like a ſpear, darting its rays from weſt to eaſt. When the ſun was ſunk below the horizon, by the luſtre of its own rays it enlightened all the borders of the earth, not permitting the other ſtars to ſhew their light, or the ſhades of night to darken the air, becauſe its light exceeded that of the others, and extended itſelf to the upper part of the heavens, flaming," &c. *Hiſt. Byzant. Duc. Mich. Nepot.* From the ſituation of the tail of this comet, and the time of its firſt appearance, we may infer that the head was then near the ſun, and went farther from him every day; for that comet continued three months. In the year 1527, *Aug.* 11, about four in the morning, there was ſeen almoſt throughout *Europe* a terrible comet in *Leo,* which continued flaming an hour and a quarter every day. It roſe from the eaſt, and aſcended to the ſouth and weſt to a prodigious length. It was moſt conſpicuous to the north, and its cloud (that is, its tail) was very terrible; having, according to the fancies of the vulgar, the form of an arm a little bent holding a ſword of a vaſt magnitude. In the year 1618, in the end of *November,* there began a rumour, that there appeared about ſun-riſing a bright beam, which was the tail of a comet whoſe head was yet concealed within the brightneſs of the ſolar rays. On *Nov.* 24, and from that time, the comet itſelf appeared with a bright light, its head and tail being extremely reſplendent. The length of the tail, which was at firſt 20 or 30 deg.

increaſed till *December* 9, when it aroſe to 75 deg. but with a light much more faint and dilute than at the beginning. In the year 1668, *March* 5, N. S. about ſeven in the evening, *P. Valent. Eſtancius,* being in *Braſil,* ſaw a comet near the horizon in the ſouth-weſt. Its head was ſmall, and ſcarcely diſcernible, but its tail extremely bright and refulgent, ſo that the reflection of it from the ſea was eaſily ſeen by thoſe who ſtood upon the ſhore. This great ſplendor laſted but three days, decreaſing very remarkably from that time. The tail at the beginning extended itſelf from weſt to ſouth, and in a ſituation almoſt parallel to the horizon, appearing like a ſhining beam 23 deg. in length. Afterwards, the light decreaſing, its magnitude increaſed till the comet ceaſed to be viſible; ſo that *Caſſini,* at *Bologna,* ſaw it (*Mar.* 10. 11, 12) riſing from the horizon 32 deg. in length. In *Portugal* it is ſaid to have taken up a fourth part of the heavens (that is, 45 deg.), extending itſelf from weſt to eaſt with a notable brightneſs; though the whole of it was not ſeen, becauſe the head in this part of the world always lay hid below the horizon. From the increaſe of the tail it is plain that the head receded from the ſun, and was neareſt to it at the beginning, when the tail appeared brighteſt.

To all theſe we may add the comet of 1680, whoſe wonderful ſplendor at the conjunction of the head with the ſun was above deſcribed. But ſo great a ſplendor argues the comets of this kind to have really paſſed near the fountain of light, eſpecially ſince the tails never ſhine ſo much in their oppoſition to the ſun; nor do we read that fiery beams have ever appeared there.

Laſtly, the ſame thing is inferred (p. 259 to 261) from the light of the heads increaſing in the receſs of the comets from the earth towards the ſun, and decreaſing in their return from the ſun towards the earth; for ſo the laſt comet of the year 1665 (by the obſervation of *Hevelius*), from the time that it was firſt ſeen, was always loſing of its apparent motion, and therefore had already paſſed its perigee; yet the ſplendor of its head was daily increaſing, till, being hid by the ſun's rays, the comet ceaſed to appear. The comet of the year 1683 (by the obſervation of the ſame *Hevelius*), about the end of *July*, when it firſt appeared, moved at a very ſlow rate, advancing only about 40 or 45 minutes in its orbit in a day's time. But from that time its diurnal motion was continually upon the increaſe till *September* 4, when it aroſe to about 5 degrees; and therefore in all this interval of time the comet was approaching to the earth. Which is likewiſe proved from the diameter of its head meaſured with a micrometer; for, *Auguſt* the 6th, *Hevelius* found it only 6′ 5″, including the coma; which, *September* 2, he obſerved 9′ 7″. And therefore its head appeared far leſs about the beginning than towards the end of its motion, though about the beginning, becauſe nearer to the ſun, it appeared far more lucid than towards the end, as the ſame *Hevelius* declares. Wherefore in all this interval of time, on account of its receſs from the ſun, it decreaſed in ſplendor, notwithſtanding its acceſs towards the earth. The comet of the year 1618, about the middle of *December*, and that of the year 1680, about the end of the ſame month, did both move with their greateſt velocity, and were therefore then in their perigees; but the greateſt ſplendor of their heads was ſeen two weeks before, when they had juſt got clear of the ſun's rays; and the greateſt ſplendor of their tails a little more early, when yet nearer to the ſun. The head of the former comet, according to the obſervations of *Cyſatus*, *Dec.* 1, appeared greater than the ſtars of the firſt magnitude; and, *Dec.* 16 (being then in its perigee), of a ſmall magnitude, and the ſplendor or clearneſs was much diminiſhed. *Jan.* 7, *Kepler*, being uncertain about the head, left off obſerving. *Dec.* 12, the head of the laſt comet was ſeen and obſerved by *Flamſted* at the diſtance of 9 degrees from the ſun, which a ſtar of the third magnitude could hardly have been. *December* 15 and 17, the ſame appeared like a ſtar of the third magnitude, its ſplendor being diminiſhed by the bright clouds near the ſetting ſun. *Dec.* 26, when it moved with the greateſt ſwiftneſs, and was almoſt in its perigee, it was inferior to *Os Pegaſi*, a ſtar of the third magnitude. *Jan.* 3, it appeared like a ſtar of the fourth; *Jan.* 9, like a ſtar of the fifth. *Jan.* 13, it diſappear-ed, by reaſon of the brightneſs of the moon, which was then in its increaſe. *Jan.* 25, it was ſcarcely equal to the ſtars of the ſeventh magnitude. If we take equal times on each hand of the perigee, the heads placed at remote diſtances would have ſhined equally before and after, becauſe of their equal diſtances from the earth. That in one caſe they ſhined very bright, and in the other vaniſhed, is to be aſcribed to the nearneſs of the ſun in the firſt caſe, and his diſtance in the other; and from the great difference of the light in theſe two caſes we infer its great nearneſs in the firſt of them; for the light of the comets uſes to be regular, and to appear greateſt when their heads move the ſwifteſt, and are therefore in their perigees; excepting in ſo far as it is increaſed by their nearneſs to the ſun.

From theſe things I at laſt diſcovered why the comets frequent ſo much the region of the ſun. If they were to be ſeen in the regions a great way beyond Saturn, they muſt appear oftener in thoſe parts of the heavens that are oppoſite to the ſun; for thoſe which are in that ſituation would be nearer to the earth, and the interpoſition of the ſun would obſcure the others: but, looking over the hiſtory of comets, I find that four or five times more have been ſeen in the hemiſphere toward the ſun than in the oppoſite hemiſphere; beſides, without doubt, not a few which have been hid by the light of the ſun; for comets deſcending into our parts neither emit tails, nor are ſo well illuminated by the ſun, as to diſcover themſelves to our naked eyes, till they are come nearer to us than Jupiter. But the far greater part of that ſpherical ſpace, which is deſcribed about the ſun with ſo ſmall an interval, lies on that ſide of the earth which regards the ſun, and the comets in that greater part are more ſtrongly illuminated, as being for the moſt part nearer to the ſun: beſides, from the remarkable eccentricity of their orbits, it comes to paſs that their lower apſides are much nearer to the ſun than if their revolutions were performed in circles concentric to the ſun.

Hence alſo we underſtand why the tails of the comets, while their heads are deſcending towards the ſun, always appear ſhort and rare, and are ſeldom ſaid to have exceeded 15 or 20 deg. in length; but in the receſs of the heads from the ſun often ſhine like fiery beams, and ſoon after reach to 40, 50, 60, 70 deg. in length, or more. This great ſplendor and length of the tails ariſes from the heat which the ſun communicates to the comet as it paſſes near it. And thence, I think, it may be concluded, that all the comets that have had ſuch tails have paſſed very near the ſun.

Hence alſo we may collect that the tails ariſe from the atmoſpheres of the heads (p. 287 to 288): but we have had three ſeveral opinions about the tails of comets; for ſome will have it that they are nothing elſe but the beams of the ſun's light tranſmitted through the comets' heads, which they ſuppoſe to be tranſparent; others, that they proceed from the refraction which light ſuffers in paſſing from the comet's head to the earth; and, laſtly, others, that they are a ſort of clouds or vapour conſtantly riſing from the comets' heads, and tending towards the parts oppoſite to the ſun. The firſt is the opinion of ſuch as are yet unacquainted with optics; for the beams of the ſun are not ſeen in a darkened room, but in conſequence of the light that is reflected from them by the little particles of duſt and ſmoke which are always flying about in the air; and hence it is that in air impregnated with thick ſmoke they appear with greater brightneſs, and are more faintly and more difficultly ſeen in a finer air; but in the heavens, where there is no matter to reflect the light, they are not to be ſeen at all. Light is not ſeen as it is in the beams, but as it is thence reflected to our eyes; for viſion is not made but by rays falling upon the eyes, and therefore there muſt be ſome reflecting matter in thoſe parts where the tails of comets are ſeen; and ſo the argument turns upon the third opinion; for that reflecting matter can be no where found but in the place of the tail, becauſe otherwiſe, ſince all the celeſtial ſpaces are equally illuminated by the ſun's light, no part of the heavens could appear with more ſplendor than another. The ſecond opinion is liable to many difficulties. The tails of comets are never ſeen variegated with thoſe colours which ever uſe to be inſeparable from refraction; and the diſtinct tranſmiſſion of the light of the fixed ſtars and planets to us is a demonſtration that the æther or celeſtial medium is not endowed with any refractive power. For as to what is alledged that the fixed ſtars have been ſometimes ſeen by the *Egyptians* environed with a coma or capillitium, becauſe that has but rarely happened, it is rather to be aſcribed to a caſual refraction of clouds, as well as the radiation and ſcintillation of the fixed ſtars to the refractions both of the eyes and air; for upon applying a teleſcope to the eye, thoſe radiations and ſcintillations immediately diſappear. By the tremulous agitation of the air and aſcending vapours, it happens that the rays of light are alternately turned aſide from the narrow ſpace of the pupil of the eye; but no ſuch thing can have place in the much wider aperture of the object-glaſs of a teleſcope; and hence it is that a ſcintillation is occaſioned in the former caſe which ceaſes in the latter; and this ceſſation in the latter caſe is a demonſtration of the regular tranſmiſſion of light through the heavens without any ſenſible refraction. But, to obviate an objection that may be made from the appearing of no tail in ſuch comets as ſhine but with a faint light, as if the ſecondary rays were then too weak to affect the eyes, and for this reaſon it is that the tails of the fixed ſtars do not appear, we are to conſider that by the means of teleſcopes the light of the fixed ſtars may be augmented above an hundred fold, and yet no tails are ſeen; that the light of the planets is yet more copious without any tail, but that comets are ſeen ſometimes with huge tails, when the light of their heads is but faint and dull; for ſo it happened in the comet of the year 1680, when in the month of *December* it was ſcarcely equal in light to the ſtars of the ſecond magnitude, and yet emitted a notable tail, extending to the length of 40°, 50°, 60°, or 70°, and upwards; and afterwards, on the 27th and 28th of *January*, the head appeared but as a ſtar of the ſeventh magnitude; but the tail (as was ſaid above), with a light that was ſenſible enough, though faint, was ſtretched out to 6 or 7 degrees in length, and with a languiſhing light that was more difficultly ſeen, even to 12° and upwards. But on the 9th and 10th of *February*, when to the naked eye the head appeared no more, I ſaw through a teleſcope the tail of 2° in length. But farther; if the tail was owing to the refraction of the celeſtial matter, and did deviate from the oppoſition of the ſun, according as the figure of the heavens requires, that deviation, in the ſame places of the heavens, ſhould be always directed towards the ſame parts: but the comet of the year 1680, *December* 28^d. $8\frac{1}{2}^h$. P. M. at *London*, was ſeen in *Piſces* 8° 41′, with latitude north 28° 6′, while the ſun was in *Capricorn* 18° 26′. And the comet of the year 1577, *December* 29, was in *Piſces* 8° 41′, with latitude north 28° 40′; and the ſun, as before, in about *Capricorn* 18° 26′. In both caſes the ſituation of the earth was the ſame, and the comet appeared in the ſame place of the heavens; yet in the former caſe the tail of the comet (as well by my obſervations as by the obſervations of others) deviated from the oppoſition of the ſun towards the north by an angle of $4\frac{1}{2}$ degrees, whereas in the latter there was (according to the obſervation of *Tycho)* a deviation of 21 degrees towards the ſouth. The refraction, therefore, of the heavens being thus diſproved, it remains that the phænomena of the tails of comets muſt be derived from ſome reflecting matter. That vapours ſufficient to fill ſuch immenſe ſpaces may ariſe from the comets' atmoſpheres, may be eaſily underſtood by what follows.

It is well known that the air near the ſurface of our earth poſſeſſes a ſpace about 1200 times greater than water of the ſame weight; and therefore a cylindric column of air 1200 feet high is of equal weight with a cylinder of water of the ſame breadth, and but one foot high. But a cylinder of air reaching to the top of the atmoſphere is of equal weight with a cylinder of water about 33 feet high; and therefore if from the whole cylinder of air the lower part of 1200 feet high is taken away, the remaining upper part will be of equal weight with a cylinder of water 32 feet high. Wherefore at the height of 1200 feet, or two furlongs, the weight of the incumbent air is leſs, and conſequently the rarity of the compreſſed air greater, than near the ſurface of the earth in the ratio of 33 to 32. And, having this ratio, we may compute the rarity of the air in all places whatſoever (by the help of cor. prop. 2, book 2), ſuppoſing the expanſion thereof to be reciprocally proportional to its compreſſion; and this proportion has been proved by the experiments of *Hooke* and others. The reſult of the computation I have ſet down in the following table, in the firſt column of which you have the height of the air in miles, whereof 4000 make a ſemi-diameter of the earth; in the ſecond the compreſſion of the air, or the incumbent weight; in the third its rarity or expanſion, ſuppoſing gravity to decreaſe in the duplicate ratio of the diſtances from the earth's centre. And the *Latin* numeral characters are here uſed for certain numbers of ciphers, as 0,xvii 1224 for 0,000000000000000001224, and 26956 xv for 26956000000000000000.

AIR's

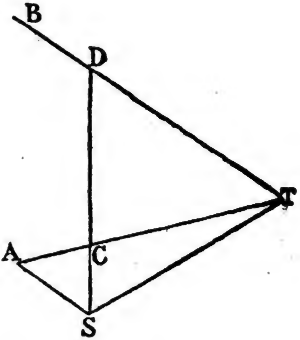
Height.	Compreſſion.	Expanſion.
0	33	1
5	17,8515	1,8486
10	9,6717	3,4151
20	2,852	11,571
40	0,2525	136,83
400	0,xvii 1224	26956 xv
4000	0,cv 4465	73907 cii
40000	0,cxcii 1628	20263 clxxxix
400000	0,ccx 7895	41798 ccvii
4000000	0,ccxii 9878	33414 ccix
Infinite.	0,ccxii 6041	54622 ccix

But from this table it appears that the air, in proceeding upwards, is rarefied in ſuch manner, that a ſphere of that air which is neareſt to the earth, of but one inch in diameter, if dilated with that rarefaction which it would have at the height of one ſemi-diameter of the earth, would fill all the planetary regions as far as the ſphere of Saturn, and a great way beyond; and at the height of ten ſemi-diameters of the earth would fill up more ſpace than is contained in the whole heavens on this ſide the fixed ſtars, according to the preceding computation of their diſtance. And though, by reaſon of the far greater thickneſs of the atmoſpheres of comets, and the great quantity of the circum-ſolar centripetal force, it may happen that the air in the celeſtial ſpaces, and in the tails of comets, is not ſo vaſtly rarefied, yet from this computation it is plain that a very ſmall quantity of air and vapour is abundantly ſufficient to produce all the appearances of the tails of comets; for that they are indeed of a very notable rarity appears from the ſhining of the ſtars through them. The atmoſphere of the earth, illuminated by the ſun's light, though but of a few miles in thickneſs, obſcures and extinguiſhes the light not only of all the ſtars, but even of the moon itſelf; whereas the ſmalleſt ſtars are ſeen to ſhine through the immenſe thickneſs of the tails of comets, likewiſe illuminated by the ſun, without the leaſt diminution of their ſplendor.

Kepler aſcribes the aſcent of the tails of comets to the atmoſpheres of their heads, and their direction towards the parts oppoſite to the ſun to the action of the rays of light carrying along with them the matter of the comets' tails; and without any great incongruity we may ſuppoſe that, in ſo free ſpaces, ſo fine a matter as that of the æther may yield to the action of the rays of the ſun's light, though thoſe rays are not able ſenſibly to move the groſs ſubſtances in our parts, which are clogged with ſo palpable a reſiſtance. Another author thinks that there may be a ſort of particles of matter endowed with a principle of levity as well as others are with a power of gravity; that the matter of the tails of comets may be of the former ſort, and that its aſcent from the ſun may be owing to its levity; but, conſidering the gravity of terreſtrial bodies is as the matter of the bodies, and therefore can be neither more nor leſs in the ſame quantity of matter, I am inclined to believe that this aſcent may rather proceed from the rarefaction of the matter of the comets' tails. The aſcent of ſmoke in a chimney is owing to the impulſe of the air with which it is entangled. The air rarefied by heat aſcends, becauſe its ſpecific gravity is diminiſhed, and in its aſcent carries along with it the ſmoke with which it is engaged. And why may not the tail of a comet riſe from the ſun after the ſame manner? for the ſun's rays do not act any way upon the mediums which they pervade but by reflection and refraction; and thoſe reflecting particles heated by this action, heat the matter of the æther which is involved with them. That matter is rarefied by the heat which it acquires, and becauſe by this rarefaction the ſpecific gravity, with which it tended towards the ſun before, is diminiſhed, it will aſcend therefrom like a ſtream, and carry along with it the reflecting particles of which the tail of the comet is compoſed; the impulſe of the ſun's light, as we have ſaid, promoting the aſcent.

But that the tails of comets do ariſe from their heads (p. 288), and tend towards the parts oppoſite to the ſun, is farther confirmed from the laws which the tails obſerve; for, lying in the planes of the comets' orbits which paſs through the ſun, they conſtantly deviate from the oppoſition of the ſun towards the parts which the comets' heads in their progreſs along thoſe orbits have left; and to a ſpectator placed in thoſe planes they appear in the parts directly oppoſite to the ſun; but as the ſpectator recedes from thoſe planes, their deviation begins to appear, and daily becomes greater. And the deviation, *cæteris paribus,* appears leſs when the tail is more oblique to the orbit of the comet, as well as when the head of the comet approaches nearer to the ſun; eſpecially if the angle of deviation is eſtimated near the head of the comet. Farther; the tails which have no deviation appear ſtraight, but the tails which deviate are likewiſe bended into a certain curvature; and this curvature is greater when the deviation is greater, and is more ſenſible when the tail, *cæteris paribus,* is longer; for in the ſhorter tails the curvature is hardly to be perceived. And the angle of deviation is leſs near the comet's head, but greater towards the other end of the tail, and that becauſe the lower ſide of the tail regards the parts from which the deviation is made, and which lie in a right line drawn out infinitely from the ſun through the comet's head. And the tails that are longer and broader, and ſhine with a ſtronger light, appear more reſplendent and more exactly defined on the convex than on the concave ſide. Upon which accounts it is plain that the phænomena of the tails of comets depend upon the motions of their heads, and by no means upon the places of the heavens in which their heads are ſeen; and that, therefore, the tails of the comets do not proceed from the refraction of the heavens, but from their own heads, which furniſh the matter that forms the tail; for as in our air the ſmoke of a heated body aſcends either perpendicularly if the body is at reſt, or obliquely if the body is moved obliquely, ſo in the heavens, where all the bodies gravitate towards the ſun, ſmoke and vapour muſt (as we have already ſaid) aſcend from the ſun, and either riſe perpendicularly if the ſmoking body is at reſt, or obliquely if the body, in the progreſs of its motion, is always leaving thoſe places from which the upper or higher parts of the vapours had riſen before. And that obliquity will be leſs where the vapour aſcends with more velocity, to wit, near the ſmoking body, when that is near the ſun; for there the force of the ſun by which the vapour aſcends is ſtronger. But becauſe the obliquity is varied, the column of vapour will be incurvated; and becauſe the vapour in the preceding ſide is ſomething more recent, that is, has aſcended ſomething more lately from the body, it will therefore be ſomething more denſe on that ſide, and muſt on that account reflect more light, as well as be better defined; the vapour on the other ſide languiſhing by degrees, and vaniſhing out of ſight.

But it is none of our preſent buſineſs to explain the cauſes of the appearances of nature. Let thoſe things which we have laſt ſaid be true or falſe, we have at leaſt made out, in the preceding diſcourſe, that the rays of light are directly propagated from the tails of comets in right lines through the heavens, in which thoſe tails appear to the ſpectators wherever placed; and conſequently the tails muſt aſcend from the heads of the comets towards the parts oppoſite to the ſun. And from this principle we may determine anew the limits of their diſtances in manner following. Let S repreſent the ſun, T the earth, STA the elongation of a comet from the ſun, and ATB the apparent length of its tail; and becauſe the light is propagated from the extremity of the tail in the direction of the right line TB, that extremity muſt lie ſomewhere in the line TB. Suppoſe it in D, and join DS cutting TA in C. Then, becauſe the tail is always ſtretched out towards the parts nearly oppoſite to the ſun, and therefore the ſun, the head of the comet, and the extremity of the tail, lie in a right line, the comet's head will be found in C. Parallel to TB draw SA,



meeting the line TA in A, and the comet's head C muſt neceſſarily be found between T and A, becauſe the extremity of the tail lies ſomewhere in the infinite line TB; and all the lines SD which can poſſibly be drawn from the point S to the line TB muſt cut the line TA ſomewhere between T and A. Wherefore the diſtance of the comet from the earth cannot exceed the interval TA, nor its diſtance from the ſun the interval SA beyond, or ST on this ſide the ſun. For inſtance; the elongation of the comet of 1680 from the ſun, *Dec.* 12, was 9°, and the length of its tail 35° at leaſt. If, therefore, a triangle TSA is made, whoſe angle T is equal to the elongation 9°, and angle A equal to ATB, or to the length of the tail, viz. 35°, then SA will be to ST, that is, the limit of the greateſt poſſible diſtance of the comet from the ſun to the ſemi-diameter of the *orbis magnus*, as the ſine of the angle T to the ſine of the angle A, that is, as about 3 to 11. And therefore the comet at that time was leſs diſtant from the ſun than by $\frac{3}{11}$ of the earth's diſtance from the ſun, and conſequently either was within the orb of Mercury, or between that orb and the earth. Again, *Dec.* 21, the elongation of the comet from the ſun was $32\frac{2}{3}°$, and the length of its tail 70°.

Wherefore as the ſine of $32\frac{2}{5}^{\circ}$ to the ſine of 70°, that is, as 4 to 7, ſo was the limit of the comet's diſtance from the ſun to the diſtance of the earth from the ſun, and conſequently the comet had not then got without the orb of Venus. *Dec.* 28, the elongation of the comet from the ſun was 55°, and the length of its tail 56°; and therefore the limit of the comet's diſtance from the ſun was not yet equal to the diſtance of the earth from the ſame, and conſequently the comet had not then got without the earth's orbit. But from its parallax we find that its egreſs from the orbit happened about *Jan.* 5, as well as that it had deſcended far within the orbit of Mercury. Let us ſuppoſe it to have been in its perihelion *Dec.* the 8th, when it was in conjunction with the ſun; and it will follow that in the journey from its perihelion to its exit out of the earth's orbit it had ſpent 28 days; and conſequently that in the 26 or 27 days following, in which it ceaſed to be farther ſeen by the naked eye, it had ſcarcely doubled its diſtance from the ſun; and by limiting the diſtances of other comets by the like arguments, we come at laſt to this concluſion,---that all comets, during the time in which they are viſible by us, are within the compaſs of a ſpherical ſpace deſcribed about the ſun as a centre, with a radius double, or at moſt triple, of the diſtance of the earth from the ſun.

And hence it follows that the comets, during the whole time of their appearance unto us, being within the ſphere of activity of the circum-ſolar force, and therefore agitated by the impulſe of that force, will (by cor. 1, prop. 13, for the ſame reaſon as the planets) be made to move in conic ſections that have one focus in the centre of the ſun, and, by radii drawn to the ſun, to deſcribe areas proportional to the times; for that force is propagated to an immenſe diſtance, and will govern the motions of bodies far beyond the orbit of Saturn.

There are three hypotheſes about comets (p. 262); for ſome will have it that they are generated and periſh as often as they appear and vaniſh; others, that they come from the regions of the fixed ſtars, and are ſeen by us in their paſſage through the ſyſtem of our planets; and, laſtly, others, that they are bodies perpetually revolving about the ſun in very eccentric orbits. In the firſt caſe, the comets, according to their different velocities, will move in conic ſections of all ſorts; in the ſecond, they will deſcribe hyperbolas, and in either of the two will frequent indifferently all quarters of the heavens, as well as thoſe about the poles as thoſe towards the ecliptic; in the third, their motions will be performed in ellipſes very eccentric, and very nearly approaching to parabolas. But (if the law of the planets is obſerved) their orbits will not much decline from the plane of the ecliptic; and, ſo far as I could hitherto obſerve, the third caſe obtains; for the comets do, indeed, chiefly frequent the zodiac, and ſcarcely ever attain to a heliocentric latitude of 40°. And that they move in orbits very nearly parabolical, I infer from their velocity; for the velocity with which a parabola is deſcribed is every where to the velocity with which a comet or planet may be revolved about the ſun in a circle at the ſame diſtance in the ſubduplicate ratio of 2 to 1 (by cor. 7, prop. 16); and, by my computation, the velocity of comets is found to be much about the ſame. I examined the thing by inferring nearly the velocities from the diſtances, and the diſtances both from the parallaxes and the phænomena of the tails, and never found the errors of exceſs or defect in the velocities greater than what might have aroſe from the errors in the diſtances collected after that manner. But I likewiſe made uſe of the reaſoning that follows.

Suppoſing the radius of the *orbis magnus* to be divided into 1000 parts: let the numbers in the firſt column of the following table repreſent the diſtance of the vertex of the parabola from the ſun's centre, expreſſed by thoſe parts; and a comet in the times expreſſed in col. 2, will paſs from its perihelion to the ſurface of the ſphere which is deſcribed about the ſun as a centre with the radius of the *orbis magnus*; and in the times expreſſed in col. 3, 4, and 5, it will double, triple, and quadruple, that its diſtance from the ſun.

TABLE I.

The diſtance of a comet's perihelion from the ſun's centre.	The time of a comet's paſſage from its perihelion to a diſtance from the ſun equal to											
	The radii of the *orbis magnus.*			To its double.			To its triple.			To its quadruple.		
	d.	h.	′	d.	h.	′	d.	h.	′	d.	h.	′
0	27	11	12	77	16	28	142	17	14	219	17	30
5	27	16	07	77	23	14						
10	27	21	00	78	06	24						
20	28	06	40	78	20	13	144	03	19	221	08	54
40	29	01	32	79	23	34						
80	30	13	25	82	04	56						
160	33	05	29	86	10	26	153	16	08	232	12	20
320	37	13	46	93	23	38						
640	37	09	49	105	01	28						
1280				106	06	35	200	06	43	297	03	46
2560							147	22	31	300	06	03

[This table, here corrected, is made on the ſuppoſition that the earth's diurnal motion is juſt 59′, and the meaſure of one minute looſely 0,2909, in reſpect of the radius 1000. If thoſe meaſures are taken true, the true numbers of the table will all come out leſs. But the difference, even when greateſt, and to the quadruple of the earth's diſtance from the ſun, amounts only to 16^{h}. 55′.]

The time of a comet's ingreſs into the ſphere of the *orbis magnus,* or of its egreſs from the ſame, may be inferred nearly from its parallax, but with more expedition by the following

TABLE II.

The apparent elongation of a comet from the ſun.	Its apparent diurnal motion in its own orbit.		Its diſtance from the earth in parts, whereof the radius of the *orbis magnus* contains 1000.
	Direct.	Retrog.	
60°	2° 18′	00° 20′	1000
65	2 33	00 35	845
70	2 55	00 57	684
72	3 07	01 09	618
74	3 23	01 25	551
76	3 43	01 45	484
78	4 10	02 12	416
80	4 57	02 49	347
82	5 45	03 47	278
84	7 18	05 20	209
86	10 27	08 19	140
88	18 37	16 39	70
90	Infinite	Infinite	00

The ingreſs of a comet into the ſphere of the *orbis magnus*, or its egreſs from the ſame, happens at the time of its elongation from the ſun, expreſſed in col. 1, againſt its diurnal motion. So in the comet of 1681, *Jan.* 4, O.S. the apparent diurnal motion in its orbit was about 3° 5′, and the correſponding elongation $71\frac{2}{3}°$; and the comet had acquired this elongation from the ſun *Jan.* 4, about ſix in the evening. Again, in the year 1680, *Nov.* 11, the diurnal motion of the comet that then appeared was about $4\frac{2}{3}°$; and the correſponding elongation $79\frac{2}{3}$ happened *Nov.* 10, a little before midnight. Now at the times named theſe comets had arrived at an equal diſtance from the ſun with the earth, and the earth was then almoſt in its perihelion. But the firſt table is fitted to the earth's mean diſtance from the ſun aſſumed of 1000 parts; and this diſtance is greater by ſuch an exceſs of ſpace as the earth might deſcribe by its annual motion in one day's time, or the comet by its motion in 16 hours. To reduce the comet to this mean diſtance of 1000 parts, we add thoſe 16 hours to the former time, and ſubduct them from the latter; and thus the former becomes *Jan.* 4^d. 10^h. afternoon; the latter *Nov.* 10, about ſix in the morning. But from the tenor and progreſs of the diurnal motions it appears that both comets were in conjunction with the ſun between *Dec.* 7 and *Dec.* 8; and from thence to *Jan.* 4^d. 10^h. afternoon on one ſide, and to *Nov.* 10^d. 6^h. of the morning on the other, there are about 28 days. And ſo many days (by Table 1) the motions in parabolic trajectories do require.

But though we have hitherto conſidered thoſe comets as two, yet, from the coincidence of their perihelions and agreement of their velocities, it is probable that in effect they were but one and the ſame; and if ſo, the orbit of this comet muſt have either been a parabola, or at leaſt a conic ſection very little differing from a parabola, and at its vertex almoſt in contact with the ſurface of the ſun. For (by Tab. 2) the diſtance of the comet from the earth, *Nov.* 10, was about 360 parts, and *Jan.* 4, about 630. From which diſtances, together with its longitudes and latitudes, we infer the diſtance of the places in which the comet was at thoſe times to have been about 280: the half of which, viz. 140, is an ordinate to the comet's orbit, cutting off a portion of its axis nearly equal to the radius of the *orbis magnus*, that is, to 1000 parts. And, therefore, dividing the ſquare of the ordinate 140 by 1000, the ſegment of the axis, we find the *latus rectum* 19,16, or in a round number 20; the fourth part whereof, 5, is the diſtance of the vertex of the orbit from the ſun's centre. But the time correſponding to the diſtance of 5 parts in Tab. 1 is 27^d. 16^h. 7′. In which time, if the comet moved in a parabolic orbit, it would have been carried from its perihelion to the ſurface of the ſphere of the *orbis magnus* deſcribed with the radius 1000, and would have ſpent the double of that time, viz. 55^d. $8\frac{1}{4}^h$. in the whole courſe of its motion within that ſphere: and ſo in fact it did; for from *Nov.* 10^d. 6^h. of the morning, the time of the comet's ingreſs into the ſphere of the *orbis magnus*, to *Jan.* 4^d. 10^h. afternoon, the time of its egreſs from the ſame, there are 55^d. 16^h. The ſmall difference of $7\frac{3}{4}^h$. in this rude way of computing is to be neglected, and perhaps may ariſe from the comet's motion being ſome ſmall matter ſlower, as it muſt have been if the true orbit in which it was carried was an ellipſis. The middle time between its ingreſs and egreſs was *December* 8^{d}. 2^{h}. of the morning; and therefore at this time the comet ought to have been in its perihelion. And accordingly that very day, juſt before ſun-riſing, Dr. *Halley* (as we ſaid) ſaw the tail ſhort and broad, but very bright, riſing perpendicularly from the horizon. From the poſition of the tail it is certain that the comet had then croſſed over the ecliptic, and got into north latitude, and therefore had paſſed by its perihelion, which lay on the other ſide of the ecliptic, though it had not yet come into conjunction with the ſun; and the comet [ſee more of this famous comet, p. 272 to 286, vol. 2] being at this time between its perihelion and its conjunction with the ſun, muſt have been in its perihelion a few hours before; for in ſo near a diſtance from the ſun it muſt have been carried with great velocity, and have apparently deſcribed almoſt half a degree every hour.

By like computations I find that the comet of 1618 entered the ſphere of the *orbis magnus December* 7, towards ſun-ſetting; but its conjunction with the ſun was *Nov.* 9, or 10, about 28 days intervening, as in the preceding comet; for from the ſize of the tail of this, in which it was equal to the preceding, it is probable that this comet likewiſe did come almoſt into a contact with the ſun. Four comets were ſeen that year, of which this was the laſt. The ſecond, which made its firſt appearance *October* 31, in the neighbourhood of the riſing ſun, and was ſoon after hid under the ſun's rays, I ſuſpect to have been the ſame with the fourth, which emerged out of the ſun's rays about *Nov.* 9. To theſe we may add the comet of 1607, which entered the ſphere of the *orbis magnus Sep.* 14, O.S. and arrived at its perihelion diſtance from the ſun about *October* 19, 35 days intervening. Its perihelion diſtance ſubtended an apparent angle at the earth of about 23 degrees, and was therefore of 390 parts. And to this number of parts about 34 days correſpond in Tab. 1. Farther; the comet of 1665 entered the ſphere of the *orbis magnus* about *March* 17, and came to its perihelion about *April* 16, 30 days intervening. Its perihelion diſtance ſubtended an angle at the earth of about ſeven degrees, and therefore was of 122 parts:

and correſponding to this number of parts, in Tab. 1, we find 30 days. Again; the comet of 1682 entered the ſphere of the *orbis magnus* about *Aug.* 11, and arrived at its perihelion about *Sep.* 16, being then diſtant from the ſun by about 350 parts, to which, in Tab. 1, belong 33½ days. Laſtly; that memorable comet of *Regiomontanus*, which in 1472 was carried through the circum-polar parts of our northern hemiſphere with ſuch rapidity, as to deſcribe 40 degrees in one day, entered the ſphere of the *orbis magnus Jan.* 21, about the time that it was paſſing by the pole, and, haſtening from thence towards the ſun, was hid under the ſun's rays about the end of *Feb.;* whence it is probable that 30 days, or a few more, were ſpent between its ingreſs into the ſphere of the *orbis magnus* and its perihelion. Nor did this comet truly move with more velocity than other comets, but owed the greatneſs of its apparent velocity to its paſſing by the earth at a near diſtance.

It appears, then, that the velocity of comets (p. 268), ſo far as it can be determined by theſe rude ways of computing, is that very velocity with which parabolas, or ellipſes near to parabolas, ought to be deſcribed; and therefore the diſtance between a comet and the ſun being given, the velocity of the comet is nearly given. And hence ariſes this problem.

PROBLEM.

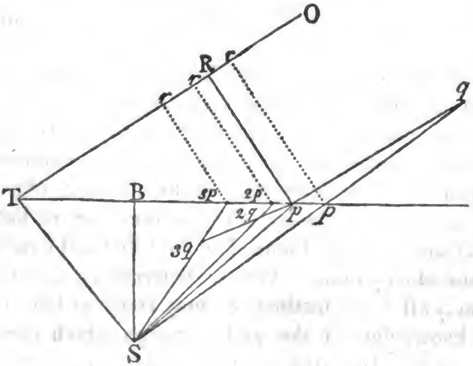
The relation betwixt the velocity of a comet and its diſtance from the ſun's centre being given, the comet's trajectory is required.

If this problem was reſolved, we ſhould thence have a method of determining the trajectories of comets to the greateſt accuracy; for if that relation be twice aſſumed, and from thence the trajectory be twice computed, and the error of each trajectory be found from obſervations, the aſſumption may be corrected by the Rule of Falſe, and a third trajectory may thence be found that will exactly agree with the obſervations. And by determining the trajectories of comets after this method, we may come, at laſt, to a more exact knowledge of the parts through which thoſe bodies travel, of the velocities with which they are carried, what ſort of trajectories they deſcribe, and what are the true magnitudes and forms of their tails according to the various diſtances of their heads from the ſun; whether, after certain intervals of time, the ſame comets do return again, and in what periods they complete their ſeveral revolutions. But the problem may be reſolved by determining, firſt, the hourly motion of a comet to a given time from three or more obſervations, and then deriving the trajectory from this motion. And thus the invention of the trajectory, depending on one obſervation, and its hourly motion at the time of this obſervation, will either confirm or diſprove itſelf; for the concluſion that is drawn from the motion only of an hour or two and a falſe hypotheſis will never agree with the motions of the comets from beginning to end. The method of the whole computation is this.

LEMMA I.

To cut two right lines OR, TP, *given in poſition, by a third right line* RP, *ſo as* TRP *may be a right angle; and, if another right line* SP *is drawn to any given point* S, *the ſolid contained under this line* SP, *and the ſquare of the right line* OR *terminated at a given point* O, *may be of a given magnitude.*

It is done by linear deſcription thus. Let the given magnitude of the ſolid be $M^2 \times N$; from any point r of the right line OR erect the perpendicular rp meeting TP in p. Then



through the point Sp draw the line Sq equal to $\frac{M^2 \times N}{Or^2}$. In like manner draw three or more right lines S2q, S3q, &c.; and a regular line q2q3q drawn through all the points q2q3q, &c. will cut the right line TP in the point P, from which the perpendicular PR is to be let fall. Q.E.F.

By trigonometry thus. Aſſuming the right line TP as found by the preceding method, the perpendiculars TR, SB, in the triangles TPR, TPS, will be thence given; and the ſide SP in the triangle SBP, as well as the error $\frac{M^2 \times N}{OR^2} -$ SP. Let this error, ſuppoſe D, be to a new error, ſuppoſe E, as the error 2p2q ± 3p3q to the error 2p3p; or as the error 2p2q ± D to the error 2pP; and this new error added to or ſubducted from the length TP, will give the correct length TP ± E. The inſpection of the figure will ſhew whether we are to add or to ſubtract; and if at any time there ſhould be uſe for a farther correction, the operation may be repeated.

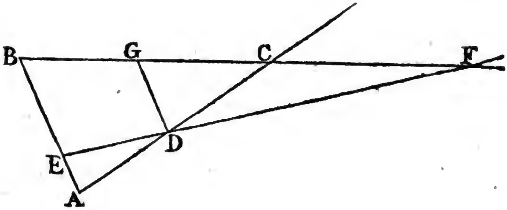
By arithmetic thus. Let us ſuppoſe the thing done, and let TP + e be the correct length of the right line TP as found out by delineation; and thence the correct lengths of the lines OR, BP, and SP, will be $OR - \frac{TR}{TP}e$, $BP + e$, and $\sqrt{SP^2 + 2BPe + ee} = \frac{M^2N}{OR^2 + \frac{2OR + TR}{TP}e + \frac{TR^2}{TP^2}ee}$. Whence, by the method of converging ſeries, we have $SP + \frac{BP}{SP}e + \frac{SB^2}{2SP^3}ee$, &c. $= \frac{M^2N}{OR^2} + \frac{2TR}{TP} \times \frac{M^2N}{OR^3}e + \frac{3TR^2}{TP^2} \times \frac{M^2N}{OR^4}ee$, &c. For the given co-efficients $\frac{M^2N}{OR^2} - SP$, $\frac{2TR}{TP} \times \frac{M^2N}{OR^3} - \frac{BP}{SP}$, $\frac{3TR^2}{TP^2} \times \frac{M^2N}{OR^4} - \frac{SB^2}{2SP^3}$, putting F, $\frac{F}{G}$, $\frac{F}{GH}$, and carefully obſerving the ſigns, we find $F + \frac{F}{G}e + \frac{F}{GH}ee = 0$, and $e + \frac{ee}{H} = -G$. Whence, neglecting the very ſmall term $\frac{e^2}{H}$, e comes out equal to — G. If the error $\frac{e^2}{H}$ is not deſpicable, take $-G-\frac{G^2}{H}=e$.

And it is to be obſerved that here a general method is hinted at for ſolving the more intricate ſort of problems, as well by trigonometry as by arithmetic, without thoſe perplexed computations and reſolutions of affected equations which hitherto have been in uſe.

LEMMA II.

To cut three right lines given in poſition by a fourth right line that ſhall paſs through a point aſſigned in any of the three, and ſo as its intercepted parts ſhall be in a given ratio one to the other.

Let AB, AC, BC, be the right lines given in poſition, and ſuppoſe D to be the given point in the line AC. Parallel to AB draw DG meeting BC in G; and, taking GF to BG in the given ratio, draw FDE; and FD will be to DE as FG to BG. Q.E.F.

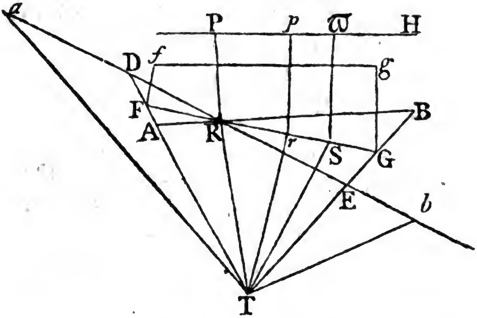


By trigonometry thus. In the triangle CGD all the angles and the ſide CD are given, and from thence its remaining ſides are found; and from the given ratios the lines GF and BE are alſo given.

LEMMA III.

To find and repreſent by a linear deſcription the hourly motion of a comet to any given time.

From obſervations of the beſt credit, let three longitudes of the comet be given, and, ſuppoſing ATR, RTB, to be their differences, let the hourly motion be required to the time of the middle obſervation TR. By *Lem.* II. draw the right line ARB, ſo as its intercepted parts AR, RB, may be as the times between the obſervations; and if we ſuppoſe a body in the whole time to deſcribe the whole line AB with an equal



motion, and to be in the mean time viewed from the place T, the apparent motion of that body about the point R will be nearly the ſame with that of the comet at the time of the obſervation TR.

The ſame more accurately.

Let Ta, Tb, be two longitudes given at a greater diſtance on one ſide and on the other; and, by *Lem.* II. draw the right line aRb ſo as its intercepted parts aR, Rb, may be as the times between the obſervations aTR, RTb. Suppoſe this to cut the lines TA, TB, in D and E; and becauſe the error of the inclination TRa increaſes nearly in the duplicate ratio of the time between the obſervations, draw FRG, ſo as either the angle DRF may be to the angle ARF, or the line DF to the line AF, in the duplicate ratio of the whole time between the obſervations aTB to the whole time between the obſervations ATB, and uſe the line thus found FG in place of the line AB found above.

It will be convenient that the angles ATR, RTB, aTA, BTb, be no leſs than of ten or fifteen degrees, the times correſponding no greater than of eight or twelve days, and the longitudes taken when the comet moves with the greateſt velocity; for thus the errors of the obſervations will bear a leſs proportion to the differences of the longitudes.

LEMMA IV.

To find the longitudes of a comet to any given times.

It is done by taking in the line FG the diſtances Rr, $R\rho$, proportional to the times, and drawing the lines Tr, $T\rho$. The way of working by trigonometry is manifeſt.

LEMMA V.

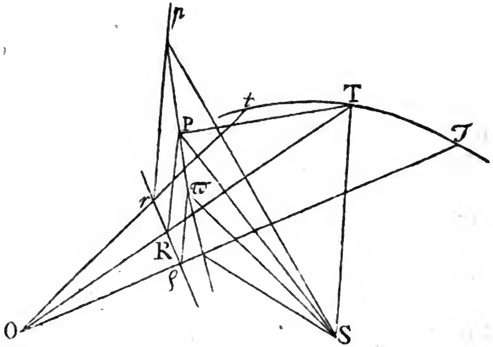
To find the latitudes.

On TF, TR, TG, as radiuſes, at right angles erect Ff, RP, Gg, tangents of the obſerved latitudes; and parallel to fg draw PH. The perpendiculars rp, $\rho\varpi$, meeting PH, will be the tangents of the ſought latitudes to Tr and $T\rho$ as radiuſes.

PROBLEM I.

From the aſſumed ratio of the velocity to determine the trajectory of a comet.

Let S repreſent the ſun; t, T, τ, three places of the earth in its orbit at equal diſtances; p, P, ϖ, as many correſponding



places of the comet in its trajectory, ſo as the diſtances interpoſed betwixt place and place may anſwer to the motion of one hour; pr, PR, $\varpi\rho$, perpendiculars let fall on the plane of the ecliptic, and $rR\rho$ the veſtige of the trajectory in this plane. Join Sp, SP, $S\varpi$, SR, ST, tr, TR, $\tau\rho$, TP; and let tr, $\tau\rho$, meet in

O, TR will nearly converge to the ſame point O, or the error will be inconſiderable. By the premiſed lemmas the angles rOR, ROϱ, are given, as well as the ratios pr to tr, PR to TR, and $\varpi\varrho$ to $\tau\varrho$. The figure $tT\tau O$ is likewiſe given both in magnitude and poſition, together with the diſtance ST, and the angles STR, PTR, STP. Let us aſſume the velocity of the comet in the place P to be to the velocity of a planet revolved about the ſun in a circle, at the ſame diſtance SP, as V to 1; and we ſhall have a line $pP\varpi$ to be determined, of this condition, that the ſpace $p\varpi$, deſcribed by the comet in two hours, may be to the ſpace $V \times t\tau$ (that is, to the ſpace which the earth deſcribes in the ſame time multiplied by the number V) in the ſubduplicate ratio of ST, the diſtance of the earth from the ſun, to SP, the diſtance of the comet from the ſun; and that the ſpace pP, deſcribed by the comet in the firſt hour, may be to the ſpace $P\varpi$, deſcribed by the comet in the ſecond hour, as the velocity in p to the velocity in P; that is, in the ſubduplicate ratio of the diſtance SP to the diſtance Sp, or in the ratio of 2Sp to SP + Sp; for in this whole work I neglect ſmall fractions that can produce no ſenſible error.

In the firſt place, then, as mathematicians, in the reſolution of affected equations, are wont, for the firſt eſſay, to aſſume the root by conjecture, ſo, in this analytical operation, I judge of the ſought diſtance TR as I beſt can by conjecture. Then, by *Lem.* II. I draw rϱ, firſt ſuppoſing rR equal to Rϱ, and again (after the ratio of SP to Sp is diſcovered) ſo as rR may be to Rϱ as 2SP to SP + Sp, and I find the ratios of the lines $p\varpi$, $r\varrho$, and OR, one to the other. Let M be to $V \times t\tau$ as OR to $p\varpi$; and becauſe the ſquare of $p\varpi$ is to the ſquare of $V \times t\tau$ as ST to SP, we ſhall have, *ex æquo*, OR^2 to M^2 as ST to SP, and therefore the ſolid $OR^2 \times SP$ equal to the given ſolid $M^2 \times ST$; whence (ſuppoſing the triangles STP, PTR, to be now placed in the ſame plane) TR, TP, SP, PR, will be given, by *Lem.* I. All this I do, firſt by delineation in a rude and haſty way; then by a new delineation with greater care; and, laſtly, by an arithmetical computation. Then I proceed to determine the poſition of the lines $r\varrho$, $p\varpi$, with the greateſt accuracy, together with the nodes and inclination of the plane $Sp\varpi$ to the plane of the ecliptic; and in that plane $Sp\varpi$ I deſcribe the trajectory in which a body let go from the place P in the direction of the given right line $p\varpi$ would be carried with a velocity that is to the velocity of the earth as $p\varpi$ to $V \times t\tau$. Q.E.F.

PROBLEM II.

To correct the aſſumed ratio of the velocity and the trajectory thence found.

Take an obſervation of the comet about the end of its appearance, or any other obſervation at a very great diſtance from the obſervations uſed before, and find the interſection of a right line drawn to the comet, in that obſervation with the plane $Sp\varpi$, as well as the comet's place in its trajectory to the time of the obſervation. If that interſection happens in this place, it is aproof that the trajectory was rightly determined; if otherwiſe, a new number V is to be aſſumed, and a new trajectory to be found; and then the place of the comet in this trajectory to the time of that probatory obſervation, and the interſection of a right line drawn to the comet with the plane of the trajectory, are to be determined as before; and by comparing the variation of the error with the variation of the other quantities, we may conclude, by the Rule of Three, how far thoſe other quantities ought to be varied or corrected, ſo as the error may become as ſmall as poſſible. And by means of theſe corrections we may have the trajectory exactly, providing the obſervations upon which the computation was founded were exact, and that we did not err much in the aſſumption of the quantity V; for if we did, the operation is to be repeated till the trajectory is exactly enough determined. Q.E.F.

End of the Syſtem of the World.

A

SHORT COMMENT

ON

SIR ISAAC NEWTON'S

PRINCIPIA.

CONTAINING

EXPLANATIONS OF SOME DIFFICULT PLACES

IN

THAT EXCELLENT WORK.

By W. Emerſon.

PREFACE.

THE PRINCIPIA being a book which is univerſally read by all the world, that pretend to any degree of philoſophical learning, it cannot be improper to explain ſuch paſſages therein as ſeem obſcure and difficult. For although it is written in as clear a ſtyle as can be done in ſo few words, yet, by reaſon of its conciſeneſs, and the difficulty of the ſubjects treated on, many things occur which require ſome farther explication, eſpecially to young beginners.

Accordingly, ſeveral mathematical writers have endeavoured to explain ſome parts or other of this Work, to make them intelligible to common readers, who, without ſuch helps, would find it very difficult to get forward.

The noble ſubjects this work treats of being no leſs than the grand fabric of the world, and the whole ſyſtem of nature, although compriſed in ſo little a compaſs, makes it highly deſerving of every illuſtration that can be given it.

The author has clearly ſhewn in this work that all the bodies in the world are actuated by the univerſal principle of gravity; which is this, that every body is attracted or impelled towards any other body by a force which is reciprocally as the ſquare of the diſtance of the two bodies. On this univerſal principle he ſhews that the motions of all the great bodies in the world are founded.

Several men had written ſyſtems of Philoſophy before Sir *Iſaac;* but, from their ignorance of nature, none of them could ſtand the teſt. But his Principles being built upon the unerring foundation of obſervations and experiments, muſt neceſſarily ſtand good, till the diſſolution of nature itſelf.

This little Treatiſe was written many years ſince; for when I ſtudied the Principia, I was frequently at a ſtop, which obliged me to make calculations here and there as I went on; and, when I had done, I ſet them down as notes upon theſe places; wherein I only meddled with thoſe places that appeared difficult to me. Theſe notes collected together are the ſubject of the following Comment; and I have reviſed the whole, and added ſeveral things that ſeemed wanting. Yet I believe there are ſome things ſtill behind, which are not ſufficiently explained by any commentator, and eſpecially ſuch as are there laid down without their demonſtrations.

As nobody is reckoned a philoſopher that does not read the Principia, therefore I thought proper to publiſh this ſmall Tract, ſuppoſing that it may be uſeful to others that have a deſire to read that celebrated Work. What I have farther to mention is this: the paſſages referred to, and which are to be explained, are not taken from the Latin edition, which would not ſuit the Engliſh reader; but from *Motte*'s tranſlation, and from the firſt edition thereof, if there happen to be more.

W. Emerſon.

ADVERTISEMENT.

Beſides the common Algebraic characters which are in uſe, I make uſe of this $\propto$, which ſignifies a proportion. Thus A $\propto$ BC, ſignifies that A is in a conſtant ratio to BC, or that A is as BC.

A

SHORT COMMENT,

&c.

[COR. 2, to the laws. And therefore if the weight p is to the weight A, &c.] For p will have the ſame effect as P, if p : P : : pH × OL : pN × perpendicular from O on pN. And P : A : : (by what went before) KO : OL. Therefore, *ex equo*, if p and A are in equilibrio, it will be p : A : : pH × KO : pN × perpendicular from O on pN. Note, the line pN ought to be drawn in fig. 2, and not PL.

[Cor. 4, ib.] This is demonſtrated in Keil's Introduction, theor. 20; as likewiſe in lem. 23, p. 86, of this Work.

[Ib. p. 20, *ſub finem*; but the diſtance between theſe two centres,] that is, between the centre of the two, and the centre of all the reſt. Farther, the actions of all the bodies may be conſidered as the ſum of the actions of every two; and then the caſe will be plain.

[Sch. to cor. 6, p. 23, then will ST repreſent] For if RV = retardation of deſcribing 2RA + 2VA, ST will be the retardation of deſcribing $\frac{1}{4}$ of that, or $\frac{1}{2}$RV + $\frac{1}{2}$VA, that is, SA or TA; and therefore the body falling from S in the air, or from T *in vacuo*, will have, nearly, the ſame velocity in A; the ſame of sA or tA; for aſcending to s in the air, or aſcending to t (or deſcending from t to A) *in vacuo* has the ſame velocity in A.

[Ib. p. 27, but if they are turned aſide by the interpoſition.] This is plain by prop. 16, Mechanics.

[Ib. And in like manner, &c.] By the ſame prop. as before.

BOOK I.

SECTION I.

[Lem. 10.] For let AE be divided into an infinite number of equal parts, any part, as D′, × by the velocity acquired in the time AD (that is, D × DB), is as the ſpace deſcribed in that little part of time D′ (for the ſpaces are as the times × by the velocities); and the ſum of all theſe products or areas, that is, the areas ADB, AEC, are as the whole ſpaces deſcribed in the times AD, AE; but theſe areas are as AD^2 and AE^2, by lem. 9.

[Ib. cor. 1. — to the bodies, and meaſured —] that is, and the ſaid errors meaſured by the diſtances of the bodies, &c.

[Lem. 11, have a finite curvature.] Theſe words exclude thoſe curves whoſe radius of curvature is infinitely ſmall, or infinitely great.

[Ib. ſchol.] All this may be univerſally demonſtrated after this manner. Let AB, AF (Fig. 1), be two paraboloids. Let *latus rectum* of AF $=$ a, of AB $=$ b, AC $=$ z, AE $=$ x, CB, EF, or AD $=$ y. And let m be any affirmative index, and ſuppoſe $a^m x = y^{m+1}$. And $b^{m+n} z = y^{m+n+1}$. Then $\frac{m}{a^{m+1}} \frac{1}{x^{m+1}} = y = \frac{m+n}{b^{m+n+1}} \times \frac{1}{z^{m+n+1}}$. And by involution, $a^m \times \overline{m+n+1}\, x^{m+n+1} = \overline{b^{m+n}} \times \overline{m+1}\, z^{m+1}$. And, therefore, $a^{m^2+mn+m} x^n : b^{m^2+mn+m+n} :: z^{m+1} : x^{m+1}$. But, becauſe x is infinitely ſmall, therefore when n is affirmative, the firſt term is infinitely leſs than the ſecond; and, therefore, z^{m+1} is infinitely leſſer than x^{m+1}; but if n is negative, the firſt term is infinitely greater than the ſecond (Fig. 1), and, therefore, z^{m+1} is infinitely greater than x^{m+1}, or z than x; that is, in caſe the firſt DB is infinitely leſs, and in caſe the ſecond infinitely greater than DF. But DB, DF, are as the curvatures of AB, AF, therefore, &c.

SECTION II.

[Pr. 1, cor. 4, as the verſed ſines, of arcs] (Fig. 2) deſcribed in equal times, tending to the centre of force, and biſect the chords; let $AB = BC$, and Bd be $\perp$ to AC, then $Ad = dC$. And when the arc AC is diminiſhed to infinity, e coincides with d; and conſequently $Ae = eC$, or BeS biſects the chord AC. Complete the parallelogram ABCf, and $eB = \frac{1}{2}fB$. Note. — He calls that the verſed ſine of an arc which is commonly called the verſed ſine of half that arc.

[Pr. 4, cor. 2.] For the forces $\propto \frac{\text{velo.}^2}{\text{rad.}} \propto \frac{\text{vel.}^2 \times \text{rad.}}{\text{rad}} \propto \frac{\text{rad.}}{\text{p times}^2}$; all the other corollaries (except the laſt) depend on this.

[Ib. cor 8.] Let $R =$ radius of curvature, $D =$ diſtance from the centre of force. Then will the p. time $\propto R^n \propto D^n$ (by ſimilar poſition), and the areas $\propto$ velocities $\propto R^{n-1} \propto D^{n-1}$. And therefore the force $\propto \frac{1}{D^{2n-1}}$; and the contrary.

[Ibid, cor. 9.] Take the arc Bd infinitely ſmall, and let BF be deſcribed by the revolving body (Fig. 3). in the ſame time that a body falls from B to E by an uniform centripetal force which it has at B; then, cd is the ſpace fallen in the time of deſcribing Bd. But Cd or Bn : BE : : (as the ſquares of the times, that is, as) $Bd^2 : BF^2 :: \frac{Bd^2}{BA} : \frac{BF^2}{BA}$. But $Bn = \frac{Bd^2}{BA}$. Therefore $BE = \frac{BF^2}{BA}$. And $BE : BF : BA$ ∺.

[Ib. ſchol. as the ſquare of the length applied to the radius;] for the number of reflections is $\propto$ velocity or length directly, and the radius reciprocally.

[Pr. 7, cor. 2, in the ſame periodical time] Let ac be $\parallel$ to RP, and da $\parallel$ to PS (Fig. 4). Then if the periodic times be equal, the areas generated in a given infinitely ſmall time muſt be equal, that is, the velocities round R and S muſt be reciprocally as (the $\perp$s on PG from R and S, that is, as) RP and SG. And ſuppoſing a, c, d, to coincide in P, the force round R to the force round S is in the complicate ratio of (of ac to ad, or) SG to SP, and the ſquares of the times of deſcribing a given arc, that is reciprocally as the ſquares of the velocities, that is, as SG^2 to RP^2. Therefore the force round R to the force round S : : is as SG^3 to $SP \times RP^2$, when the periodical times are equal.

Or thus : let p be the place of the body when the tangent pg is ‖ to the line RS. Then the velocities round R and S, in the place p, will be equal ; for the ſmall areas are equal, and their heights are equal, by reaſon of the parallels RS, pg. Draw TV, tv, and then by ſimilar triangles $Sp^3 \times pv^3 = Sg^3 \times pt^3$. And $SP^3 \times PV^3 = SG^3 \times PT^3$. Then by this prop. force round S, in P : force round S, in p : : $Sp^2 \times pv^3$ or $\frac{Sg^3 \times pt^3}{Sp}$: $SP^2 \times PV^3$ or $\frac{SG^3 \times PT^3}{SP}$.

And force round S, in p : force round R, in p : : Sp : Rp or Sg.

Alſo force round R, in p : force round R, in P : : $RP^2 \times PT^3$: $Rp^2 \times pt^3$ or $Sg^2 \times pt^3$: : *ex equo*, force round S, in P : force round R, in P : : $RP^2 \times PT^3$: $\frac{SG^3 \times PT^3}{SP}$: : $SP \times RP^2$: SG^3.

[Cor. 3, in the ſame periodic time] for then the infinitely ſmall and equal areas will be deſcribed in equal times in P, and both theſe areas and the forces will be the ſame as in a circle of the ſame curvature with the orbit at P, and therefore the forces are the ſame as in the foregoing corol.

[Pr. 8, ſch.] let ApD (Fig. 5) be an ellipſis, AP a circle. Then rn : Rn : : (pm : Pm : :) qn : Qu. And by diviſion, qr : QR : : (qn : Qn : :) CD : CA. The force in the ellipſis $\propto \frac{qt^2 \times Sp^2}{qr}$ reciprocally $=$ (becauſe $qr = \frac{QR \times CD}{CA}$) $\frac{QT^2 \times CA \times Sp^2}{QR \times CD} =$ (becauſe $\frac{QT^2}{QR} = \frac{2Pm^3}{CA^2}$) $\frac{2Pm^3 \times Sp^2}{CA \times CD}$ $=$ (becauſe $Pm^3 = \frac{pm^3 \times CA^3}{CD^3}$) $\frac{2pm^3 \times CA^2 \times Sp^2}{CD^4}$. Therefore (becauſe CA, Sp, and CD are given) the force in the ellipſis is $\propto pm^3$ reciprocally. But in the hyperbola and parabola (where CA is negative or infinite) theſe lines are ſtill given, and therefore the force in any conic ſection is reciprocally as mp^3.

[Pr. 9, will be changed —] that is $\frac{QT^2}{QR}$ is every where the ſame ratio, viz. $\propto$ SP.

[Ib. ſecond way]. For PV is (by reaſon of the given angle at P) as the radius of curvature, that is (by reaſon of the ſimilarity of the parts of the figure PQ), as SP.

[Prop. 10, ſecond way. Add the rectangle uPv] for $Qv^2 + uPv = Qv^2 + \overline{TP + Tv} \times \overline{TP - Tv} = Qv^2 + TP^2 - Tv^2 = QT^2 + TP^2 =$ ſquare of the chord QP; and $Pv \times uV + uPv = VPv$.

And if the line QV be drawn, and a circle through the points PQV; the triangles PQv and PQV will be ſimilar (the $\angle QVP$ being $= QPR = PQv$), and therefore $Pv : PQ : PV$ ∺.

[Prop. 10, ſch. in the ratio of the diſtances from the centre] for the fluxion of the ordinates is augmented or diminiſhed in the ſame ratio, and that is as the force.

SECTION III.

[Prop. 13, cor. 2]. For, in the demonſtrations of prop. 11, 12, and 13, QT^2 is always equal to $QR \times$ *latus rectum.*

[Prop. 16, cor.] The four firſt corollaries are general, and agree to all conic ſections; the ſixth corollary belongs to one and the ſame parabola.

[Ib. cor. 6, it is more variable] that is, in the ellipſis, the ratio of the velocity at a leſs and greater diſtance is greater than the ratio of the ſquare roots of the greater and leſſer diſtance: in the hyperbola it is greater; for the ratio of the greater and leſſer perpendiculars (which is the ſame with this ratio of the velocities) is greater than the ratio of the ſquare roots of the greater and leſſer diſtances in the ellipſis, and leſſer than it in the hyperbola; for in the ellipſis the perpendiculars in the greateſt and leaſt diſtances are the ſame with theſe diſtances; and in the hyperbola the greateſt perpendicular poſſible is that from the focus on the aſymptote, and the leaſt the diſtance to the vertex; alſo (by Conics, prop. 24, ellipſis) the perpendicular SY $\propto \sqrt{\frac{SP}{HP}}$ (Fig. 3, ſee alſo cor. 7).

[Prop. 17, yet greater velocity] then PH (and PK) will be negative.

[Ib. cor. 2] (Fig. 5) The velocity in a circle is = velocity acquired by falling through $\frac{1}{2}DS$, by the given centripetal force.

SECTION IV.

[Prop. 18, 19, 20.] Two given points or right lines.

[Prop. 20, caſe 4. But becauſe of the ſimilar triangles VSH, vsh]; for (by ſimilar Δs, SVP, shq) SV : SP : : sh : sq : : (by ſimilar Δs, svh, spq) sv : sp. And by the ſimilar Δs SHP, shp) SP : SH : : sp : sh ; *ergo* SV : SH : : sv : sh ; and the angle VSH = psq = vsh.

[Prop. 21] three—lines. If three tangents be given, you have three points Y, from which three equal right lines as YH are to be drawn to H, by caſe 3 of the laſt lemma. If two tangents and a point P, there will be given two points Y, from which two equal lines are to be drawn to the focus H, and a third point P, from which PH is to be drawn; by caſe 2, lem. In the hyperbola, it is PH — YH = SP. If three points P be given, it is done by caſe 1, lem. 16.

SECTION V.

[Lem. 17, in given angles] each to each reſpectively in the ſame invariable angle, though they are not all equal.

[Lem. 18, and ſo (by ſuppoſition)—] for p and b are in the curve, and the rectangles of the lines from p and P are in the given ratio.

[Ib. ſch. if p happens to be in a right line] For if rpq : spt : : RPQ : SPT; and P be placed in pB; then by ſimilar triangles pq : PQ : : (pB : PB : :) pt : PT (Fig. 6); and rp : RP : : (Cp : CP : :) ps : PS. And by multiplying you have rpq : RPQ : : spt : SPT; which agrees with the lemma. But if P is placed out of pB, as at n; then becauſe the ratio $\frac{RnQ}{SnT}$ is greater than $\frac{RPQ}{SPT}$, it is alſo greater than $\frac{rpq}{spt}$, which is againſt the hypotheſis (Fig. 6).

If p is placed in DC, then rpq will be = 0, and therefore RPQ = 0; and P will alſo be placed in DC.

[Ib. If the two oppoſite angles] (Fig. 7) The Δs PCR, PBT are ſimilar, for the angles at S and T are right, and C = B, being on the ſame arc PD; therefore PR : PC : : PT : PB; and the triangles PBQ and SCP are ſimilar, for the angles at S, Q are right, and SCP = PBA; therefore PC : PS : : PB : PQ; and, *ex equo*, PR × PQ = PS × PT;

and, *e contra*, if $PR \times PQ = PS \times PT$, the locus of the point P is a circle.

And if theſe lines are not perpendicular, yet, ſince their lengths will be reciprocally as the ſines of the angles, it will follow that $PQ \times PR : PS \times PI : :$ ſine of S × S of T : ſine of Q × ſine of R, and the contrary.

[Ib. and one or two] (Fig. 8) If the point B be ſuppoſed to move towards A, and at laſt to coincide with it, ſo that AB become a tangent at A, the lemma will ſtill hold. And if B paſs beyond A, then the figure will be converted into this BECDEAB; the lemma will ſtill hold as before: and if B move to an infinite diſtance, then DB, AB will be parallels; and C alſo, then DC, AC will alſo be parallel; and alſo the conic ſection paſſing through A and D will paſs on infinitely towards C and B.

[Lem. 19.] This may be reſolved as prob. 12, in the Univerſal Arithmetic.

[Lem. 20.] Here is another figure relating to this lemma (Fig. 9).

[Lem. 21. Therefore (by lem. 20) the point D] for the angles CPR, BPT, CPB are given, and therefore the lines PT, PR (to which the ſides AQ, AS of the parallelogram are parallel) are given by poſition (See fig. above).

Here is another figure of lem. 21 (Fig. 10). This lemma is the ſame with prob. 53, of the Univerſal Arithmetic.

[Prop. 22.] (Fig. 11) This is the ſame with prob. 55, in the author's Univerſal Arithmetic.

[Prop. 23, caſe 1.] This is the ſame with prob. 56, of the Univerſal Arithmetic, or prob. 57.

[Prop. 23 (Fig. 11), caſe 2. HA^2 will become to AI^2] for let n be a point in the conic ſection infinitely near a, through both which the line ih paſſes; then ih is a tangent at a. Then (cor. 4, prop. 44, b. I. my Conics) it will be, $bgd : bhd : : pgc : xhy = \frac{bhd \times pgc}{bgd}$, and hna, or ha^2 : ian, or $ia^2 : : xhy$, or $\frac{bhd \times pgc}{bgd} : pic : : bhd \times pgc : pic \times bgd$.

[Prop. 24, — will (by the properties of the conic ſections)] (Fig. 12) becauſe any tangent may be ſuppoſed to cut the curve in two points infinitely near each other, therefore (by cor. 4, prop. 44, b. I. Ellipſis) theſe proportions follow. Alſo draw kw ∥ to ga, to interſect pg, and let $nk^2 = tkw$. Then tkw or $nk^2 : ha^2 :: dkb : bhd :: kr^2 : hr^2$. And nk : ha :: kr : hr. And n, r, a, fall in one right line. Again, kn : ga :: $kp^2 : gp^2$; and kn : gn :: kp : gp. Whence n, p, a fall in one right line. And therefore the points p, r, a, are in one right line.

[Lem. 22. Thus any right lines converging] Let the lines be kg, kh (Fig. 13). Draw OB, OeD, and pe ∥ to nB. Let m, q, be the projected points of f and h. Then (becauſe in the point B, OB = OD) mn (or Bn — Bm) = fr. Alſo qp (or ep — eq) : hg (or Dg — Dh) :: Oe : OD :: AB : AD :: fr, or mn : hg. Therefore qp = mn, therefore pn, qm (which are the projected lines of gr, hf), are parallel.

[Ib. we ſhall have the ſolution required]. (Fig. 13) For the figure hgi ſuppoſed now to be given, may be transformed into the firſt figure HGI, by making as Od to dg, ſo is OD to DG parallel to the radius AO.

[Ib. For as often as two conic ſections] For theſe conic ſections being transformed into ſimpler ones, give the point of interſection; and thereby is had an ordinate drawn from that point of interſection in the transformed curves, correſponding to the interſection of the given curves.

[Prop. 25.] Let KG, KH, be (Fig. 14) two tangents meeting in K, yx the third tangent meeting ba in y. Let the lines KH, KG, be projected in ki, lh; and yb, yx, into kl, ih; then you have the parallelogram hikl; then proceed according to the propoſition.

[Ib. For by the properties of the conic ſections] (by cor. 4, prop. 44, Conics 1).

[Ib. But according as the points.] (Fig. 15) This is plain from the nature of the ellipſis and (Fig. 16) hyperbola; and the figure cannot be a parabola, by reaſon of two parallel tangents ih and kl.

[Lem. 23.] The lemma is univerſal, as will appear by applying the demonſtration to this figure 17.

[Lem. 24, from the nature of the conic ſections] (by prop. 46, b. I. of my Conic Sections).

[Ib. cor. 1.] This holds as well (Fig. 18) when the tangent FG is on the other ſide.

[Lem. 25, alſo KH is to HL] (by cor. 1, lem. 24) for the tangents FH, LH, cut the parallel tangents ML, IK, in F and K.

[Lem. 25, cor. 2.] This (Fig. 19) holds as well when qe is on the other ſide of the figure; for in all caſes (by cor. 1) KQ × ME is given wherever the points Q, E fall, as ſuppoſe in q, e; for(by cor. 1, lem. 24) Bq : AM, or BK : : eI : eM; and, by diviſion, Kq : BK : : MI : eM. And Kq × eM = BK × MI = (by the prop.) KQ × ME; and KQ : Me : : (Kq : ME : :) Qq : Ee.

[Ib. cor. 3.] for ſince eM : ME : : QK : Kq; therefore (by lemma 23) if the right lines eQ, MK, and Eq, be drawn, the points of biſection will be placed in a right line given in poſition.

[Prop. 27, ſch. deſcribe the circle BKGC.] for the angle BKC (which is equal to the ſum of the given angles PBK and KCP) is always given.

[Ib. and when thoſe other legs CK, BK] For it muſt be obſerved, that when the lines BP, CP, touch the curve at an infinite diſtance, that theſe lines are parallel to one another, and to the aſymptote. Then, to know the poſition of the aſymptote, as the lines Bk, Ck revolve round the circle, the interſection k will ſometimes fall into the line MN, as at the point N; then BN is parallel to one aſymptote; for the interſection muſt neceſſarily fall in the line MN. And for the ſame reaſon the line BM will be parallel to the other aſymptote. And, therefore, if the angle between them be biſected by the line OH (which is done by the perp. OH), then that line is the greater axis, or parallel to it.

[Ib. ſch. But the ſquares of the axes] For the angle NBM = angle between the aſymptotes = NLM, and NLH = half the angle of the aſymptotes. Therefore LH is to HN as the tranſverſe to the conjugate. And the ſquares of theſe axes are as LH^2 to HN^2, or as HN^2 to HK^2, that is, as LH to HK.

[Ib. There are alſo other lemmas] For if the ſections (Fig. 20) are ſimilar, and in ſimilar poſition, and concentric, the tangent acb in c is parallel to the tangent dxe in x, and therefore ab the ordinate is biſected in c the point of contact.

[Lem. 26, cor.] This is the ſame with prob. 32, of the Univerſal Arithmetic.

[Lem. 27, cor. in the conſtruction] for then IH : HF : : (iX : XY : :) ih : hf. And IH : HG : : (iL : LM : :) ih : hg. Or, if it be made as iL : : LM : : IH : HG, it will be ih : hf : : IH : HF; therefore, on the contrary, if it be ih : hf : : (iE : EV : : iX : XY : :) IH : HF, it will be iL : LM : : IH : HG, which comes to the former conſtruction. For the ſolution of prop. 22, 23, 24, 25, 26, 27, ſee prop. 70, 71, 72, 73, 74, 75, b. III. my Conic Sections.

[Prop. 29, ſch.] Make alſo KA to AS (Fig. 21), and LT to AT, as HG to GF, and draw MS, NA. Then the figures SAKM and ATLN are ſimilar to FGHI; and ſince three of the angles S, A, K, or A, T, L, are in the proper lines CB, ED, DB, if the fourth angle M or N was in the fourth line EC, the problem would be rightly conſtructed. Therefore it is plain its place can be no where but where the line MN interſects EC, as at i, which is the place of the angle I.

Now we are to prove that PQ cuts BA in f, where F is to be placed. The triangle FGI is ſimilar to PEi (by conſtruction), and ſuppoſe them ſimilar to fgi; then the triangles Pfi and Egi are alſo ſimilar; for the angles at i are equal, and the ſides about theſe angles proportional; therefore the angle Egi = Pfi; and ſince goi = Qof, oQf or PQE will be = fig. So that to have fig ſimilar to FIG, PQF muſt interſect Eg in Q, to make the < PQE = FIG; and the reſt follows of courſe.

SECTION VI.

[Prop. 30, cor. 1.] For the times are as the areas, that is, as $\frac{4}{3}$GH × AS to $\frac{2}{3}$ × AS × 2AS.

[Ib. cor. 2.] For wherever the point P falls, viz. infinitely near A, ſtill $\frac{4}{3}$GH × AS (= area APS) = $\frac{1}{2}$AP × AS. And 8GH = 3AP. Therefore GH or the velocity of H : AP, or the velocity of P : : 3 : 8. But the velocity of H is every where the ſame, for always $\frac{4}{3}$GH × AS = area ASP. And GH ∝ ASP ∝ time of deſcribing AP.

[Ib. cor. 3.] For AP is the chord of a circle paſſing through A, S, P, and whoſe centre is H.

[Prop. 31, as GK the difference] For when F comes to touch the line GH, the point A will be diſtant towards G from the line FQO (which will then be ⊥ to GH) by the ſine of AOR or AOQ, and is then at L; therefore GK = arc GF — ſine of the arc AQ. And GK is as the time, and ſo it was in the conſtruction; therefore the point P is rightly found.

[Ib. ſchol. but ſince] This is demonſtrated in Keil's Aſtron. Lectures, p. 289, 297 (Fig. 22), or thus: Let AON = N (Fig. 23). Since (AB : SH : :) OQ : OS : : 57,29578 : B, therefore B = OS in degrees of the circle AQ. And ſince (rad. : ſine of AOQ : :) SO : SF : : B : D, then will SF = D in degrees. Let q be the true place of the body; Q the aſſumed place. Now ſince the time is as the area SAQ = OAQ + SOQ = $\overline{QA + SF} \times \frac{OA}{2}$; therefore the time is as AQ ± SF; and, therefore, nearly as AQ ± D, but accurately as Aq ± SE. Take Nϕ = D. Now OE : OQ : : (LE or SE — SF or) Nq ± Nϕ, or qϕ or Qϕ — qQ : Qq. And Qϕ : Qq : : QE : OQ, by compoſition, and becauſe QE = OQ ± OE. But, by conſtruction, OQ = $\frac{OS \times L}{R}$. And rad. (R) : coſ. of AOQ : : SO : OF or OE, therefore OE = $\frac{SO \times \text{coſ. } AOQ}{R}$. Wherefore Q$\phi$: Qq : : (QE =) $\frac{OS \times L}{R} \pm \frac{SO \times \text{coſ. } AQ}{R} : \frac{OS \times L}{R}$: : L ± coſ. AQ : L. But N — AOQ + D = (AN — AQ + Nϕ =) Qϕ. Therefore Qq = E, and AOq = AOQ + QOq = AOQ + E, nearly; and therefore E is rightly found. And, repeating the ſame work with theſe new angles, there will be found the angles F, G, H, I.

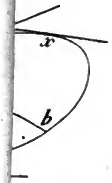
[Ib. the area AIKP will be given] (Fig. 23) by prop. 87, cor 2, and ſchol. my Conic Sections, b. II. Alſo AIKP = OPA, by cor. 1, prop. 86, ib.

Alſo area deſcribed $= ASQ = ASP + PSQ = ASP + PQ \times \frac{1}{2}SN$ (nearly) $= A$ (by ſuppoſition); therefore $A - ASP = \frac{1}{2}SN \times PQ$; and $PQ = \frac{2A - 2ASP}{SN}$, nearly.

[Ib. and by ſuch computations.—But the particular (Fig. 24) calculus] with the radius 1 and centre H deſcribe the circle RSE; and draw SP, Sp, HPT, Hpf. Let fall the perpendiculars pt, pu, upon the lines HP, SP; which will be equal, becauſe the angles pPt (HPB) and pPn are equal.

Let the arc $RT = z$, ſine $TQ = s$, coſ. $HQ = x$, $SP = y$, $HP = v$, AO or $OB = a$, SO or $OH = n$, $OD = c$, $T =$ mean anomaly, $l =$ *latus rectum.*

By ſimilar ſectors, HT (t) : HP (v) : : Tf ($\dot{z}$) : pt or pn $= v\dot{z}$. And the area $\frac{SP \times pu}{2} = \frac{vy\dot{z}}{2} =$ fluxion of BPS. But (cor. 2, prop. 23, ellipſis) $vy = cc + \frac{nn}{cc} \times PM^2$, and (cor. 3, prop. 72, ib.) HP or $v = \frac{cc}{a + nx}$, and rad. (1) : PH $\left(\frac{cc}{a + nx}\right)$: : s : $PM = \frac{ccs}{a + nx}$, therefore, $vy = cc + \frac{nnccss}{\overline{a + nx}^2}$, and area $SPp = \frac{1}{2}cc\dot{z} + \frac{\frac{1}{2}nnccss\dot{z}}{aa + 2anx, \&c.} = \frac{1}{2}cc\dot{z} + \frac{nnccss\dot{z}}{2aa} - \frac{n^3c^2s^2x\dot{z}}{a^3}$. But $s\dot{z} = -\dot{x}$, and $x\dot{z} = \dot{s}$, and $\frac{cc}{a} = \frac{1}{2}l$; therefore the area $SPp = \frac{1}{4}al\dot{z} - \frac{\frac{1}{4}nnls\dot{x}}{a} - \frac{n^3lss\dot{s}}{2aa}$, and the fluent $BSP = \frac{alz}{4} - \frac{nnl}{4a} \times$ area $EHQT - \frac{n^3ls^3}{6aa}$; and, corrected, $BSP = \frac{alz}{4} + \frac{nnl}{4a} \times$ area $RTQ - \frac{n^3ls^3}{6aa}$. But $RTQ = \frac{1}{2}z - \frac{1}{2}xs$. Whence $BSP = \frac{1}{4}alz + \frac{\frac{1}{4}nnl}{a} \times \overline{\frac{1}{2}z - \frac{1}{2}xs} - \frac{n^3ls^3}{6aa} = \frac{2aa + nn}{8a}lz - \frac{nnls}{aa} \times \overline{\frac{ax}{8} + \frac{nss}{6}}$.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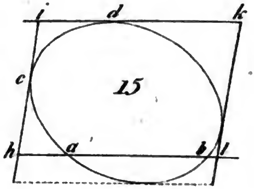
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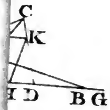
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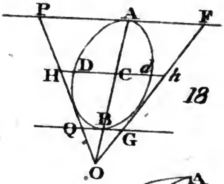
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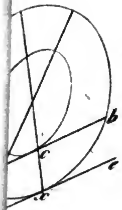
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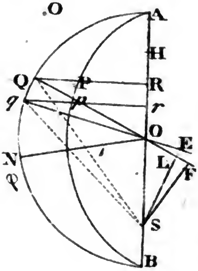
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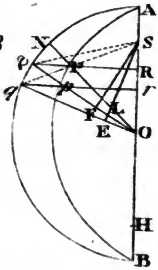
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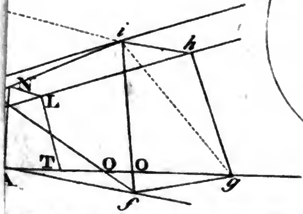
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And dividing, $z - \frac{8nns}{2a^3 + ann} \times \overline{\frac{ax}{8} + \frac{nss}{6}} = T$, the mean anomaly. Whence $z = T + \frac{nn}{4aa + 2nn} \times S.2T + \frac{4n^3}{6a^3 + 3ann} \times \overline{S.T}^3$, becauſe T is nearly = z, and 2xs = S.2T. But ſince n is very ſmall by ſuppoſition, $z = T + \frac{nn}{4aa} \times S.2T + \frac{2n^3}{3a^3} \times \overline{S.T}^3$. Where the quantities $\frac{nn}{4aa} \times S.2T$, and $\frac{2n^3}{3a^3} \times \overline{S.T}|^3$ are ſmall arcs to be added to T.

Now $D = c - \frac{cc}{a} = \frac{c}{a} \times \overline{a - c}$; and $D \times \overline{AO + OD} = \frac{c}{a} \times \overline{aa - cc} = \frac{cnn}{a} = nn$ nearly, becauſe c = a, very near. Therefore s.Y : rad. (1) : : nn : 4aa, and s.Y or Y (in ſmall arcs) $= \frac{nn}{4aa}$; alſo V : Y : : ST : rad. (1), and $V = Y \times S.2T = \frac{nn}{4aa} \times S.2T$, which is our firſt term, or his firſt equation.

Again; S.Z : rad. (1) : : 4nD, or $\frac{4nc}{a} \times \overline{a - c} : 3AO^2$, or 3aa, and $S.Z = \frac{4nc}{3a^3} \times \overline{a - c}$. But $\overline{a + c} \times \overline{a - c} = aa - cc = nn$, and $a - c = \frac{nn}{a + c} = \frac{nn}{2a}$ nearly; therefore $S.Z = \frac{4nc}{3a^3} \times \frac{nn}{2a} = \frac{2n^3c}{3a^4} = \frac{2n^3}{3a^3}$, nearly. Alſo $X : Z : : \overline{S.T}^3 :$ rad.(1). And $X = Z \times \overline{S.T}^3 = \frac{2n^3}{3a^3} \times \overline{S.T}^3$, which is the ſecond term, or his ſecond equation. And when T is above 90°, then S.2T and V will be negative, and $\angle BHP = T + X \pm V$.

He calls Y or $\frac{nn}{4aa}$ the greateſt firſt equation, becauſe it is greateſt when S.2T = 1, and T = 45°; and Z or $\frac{2n^3}{3a^3}$ the greateſt ſecond equation, becauſe the greateſt it can be is when $\overline{ST}^3 = 1$, or $T = 90°$.

SECTION VII.

[Prop. 33, to AB the principal ſemi-diameter] this ſhould be $\frac{1}{2}$AB.

[Ib. cor. 2] for the two firſt terms of the prop. are in a ratio of equality, and ſo the two laſt.

[Prop. 34. For (by cor. 7, prop. 16)], and cor. 6, prop. 4.

[Prop. 35. The ſame things ſuppoſed] to wit, that the ſpace CS is as the area SDE of the circle, rectangled hyperbola, or parabola.

[Prop. 37, as appears by prop. 34] and cor. 7, prop. 16.

[Prop. 38, acquire the velocity CD] (Fig. 25) by prop. 10, cor. 2. The periodical times of the ellipſes AD and AP are equal. The time of deſcribing AP is (as APS, that is, as ADS or) as AD. Let the ellipſis AP coincide with AC, and the time of deſcribing AC will ſtill be as AD. Farther; draw cd ∥ to CD; becauſe the time of deſcribing the whole ellipſes ADd, APp, are equal, therefore in equal times they deſcribe areas which are as the whole ellipſes, that is, as CD to CP, or ADS to APS. Wherefore in the time D deſcribes AD or Dd, P deſcribes AP or Pp, and (when P coincides with C) C deſcribes AC or Cc; therefore the velocity of D : to velocity of C : : is as Dd : Cc : : or as SD to CD; but SD and the velocity of D is given; therefore the velocity of C is as CD.

[Prop. 39, cor. 2, 3.] In theſe cor. the line PD is the ſpace the body would aſcend to, or fall from (to acquire the velocity it is projected with), by a uniform centripetal force, according to cor. 1.

SECTION VIII.

[Prop. 40, cor. 1.] This is evident, by ſuppoſing ITK convex towards C.

[Ib. cor. 2.] For by fluxions, let $AC = P$, $CD = A = x$ (Fig. 26). Then $DG \propto x^{n-1}$ (by hyp. and prop. 39), and $DEFG =$ fluxion of the area $\propto x^{n-1}\dot{x}$. And the area

DChG $\propto \frac{x^n}{n}$. And when x becomes $= p$, the area AChB $\propto \frac{p^n}{n}$. Therefore ADGB $\propto \frac{p^n - x^n}{n}$. And, therefore, by prop. 39, the velocity in D, at the diſtance A or x, is $\propto \sqrt{\frac{p^n - x^n}{n}}$, or (becauſe n is given, and $x = A$) as $\sqrt{P^n - A^n}$.

N. B. If the force is reciprocally as the diſtance, the curve BGh will be a rectangled hyperbola to the aſymptotes AC, Ch; for DG $\propto \frac{1}{CD} =$ (ſuppoſe to) $\frac{bb}{CD}$. And DG $\times$ CD $= bb$, which is the known property of the hyperbola. And the velocity at C will be infinite, for the area ABhC is infinite. And the velocity at any place D is as the hyperbolic area ABGD, which may be found by Stone's Fluxions, p. 54; or by cor. 2, prop. 87, book II. my Conic Sections.

[Prop. 41. — in the trajectories found.] One being given to find the other.

[Ib. given the circle VR] that is, CV is known.

[Ib. in the leaſt given time,] theſe ſmall parts of time are taken conſtant and invariable.

[Ib. and the triangle ICK] that is, the triangle will be invariable.

[Ib. and ſuppoſe the magnitude of Q,] This quantity is any conſtant quantity, but unknown. Suppoſe in ſome caſe $\sqrt{ABFD} = b \times IK$, and $Z = b \times KN$. And in all caſes (by prop. 40) $\sqrt{ABFD} = b \times IK$; and in all caſes Z or $\frac{Q}{A} = b \times KN$, that is, in all caſes $Q = b \times A \times KN$; which is plain, becauſe Q and A $\times$ KN are conſtant quantities: therefore if it be once $\sqrt{ABFD} : Z :: KI : KN$, or $b \times IK : b \times KN :: KI : KN$, it will always be ſo.

In this prop. the line CE is indetermined; and if Q were known, the areas of the curves abz and acx might be known to any diſtance CE.

[Ib. cor. 3.] In Fig. 4, Newt. Let $CA = a$, $CV = r$, $CD = x$, $DF = y$, $\sqrt{aa - rr} = n$.

Then the force DF being as $\frac{1}{x^3}$, the fluxion of the area ABFD is $\frac{-\dot{x}}{x^3}$, and the fluent as $\frac{1}{2x^2}$; and corrected, the fluent is $= \frac{aa - xx}{2aaxx}$ = area ABFD. Therefore the velocity at D is as $\frac{\sqrt{aa - xx}}{ax}$. Then,

1. Suppoſe a infinite, and $x = r$, then the area $\sqrt{\overline{ABFD}}$ becomes $\frac{\sqrt{aa}}{ar} = \frac{1}{r}$. And ſince at V the orbit IV is perp. to CV, therefore IK = KN, and $\sqrt{ABFD} = Z = \frac{Q}{x}$, that is, $\frac{1}{r} = \frac{Q}{r}$, and $Q = 1$, and $Z = \frac{1}{x}$; alſo $\sqrt{\overline{ABFD}} = \frac{\sqrt{aa - xx}}{ax} = \frac{a}{ax} = \frac{1}{x}$ at any place I; and ſince $\sqrt{ABFD} \left(\frac{1}{x}\right) : Z \left(\frac{1}{x}\right) :: IK : KN$, therefore TK = KN, and IN = o; therefore in this caſe the orbit is a circle, as VXR.

2. If a be leſs than infinite, and ſince at V the orbit is perp. to CV as before, therefore IK = KN, and $\sqrt{ABFD} = \frac{Q}{r}$, that is, $\frac{\sqrt{aa - rr}}{ar} = \frac{Q}{r}$, or $\frac{n}{ar} = \frac{Q}{r}$, and $Q = \frac{n}{a}$. Whence $Z = \frac{n}{ax}$. Therefore $Db = \frac{n}{a} \times \frac{1}{\sqrt{\frac{aa - xx}{aaxx} - \frac{nn}{aaxx}}} = \frac{n}{2a} \sqrt{\frac{aaxx}{rr - xx}} = \frac{nx}{2\sqrt{rr - xx}}$. And $Dc = \frac{rr}{nx} \times Db = \frac{nrr}{2x\sqrt{rr - xx}}$, and flux. of the area VacD $= \frac{nrr\dot{x}}{2x\sqrt{rr - xx}}$ = fluxion of the ſector VCX, and the Fl. : $\frac{nrr\dot{x}}{2x\sqrt{rr - xx}}$ = ſector VCX, which is as the angle VCX.

But, putting ſemi-conjugate $= c$, CH $= Z$, the hyperbolic ſector VCR $=$ Fl. : $\frac{rc\dot{z}}{2\sqrt{zz - rr}}$, and CT $= \frac{rr}{z} = x$, and $z = \frac{rr}{x}$. Then inſtead of z and $\dot{z}$, putting their values, we ſhall have the hyp. ſector VCR $=$ Fl. : $\frac{-rrc\dot{x}}{2x\sqrt{rr - xx}} = \frac{c}{R}$ × cir. ſector VCX. Therefore when $x =$ CT, the hyp. ſector VCR : cir. ſector CVX : : c : n; that is, becauſe the angle VCX is as the ſector CVX, the angle VCX is to the hyper. ſector VCR in a given ratio. And ſince CT or x continually decreaſes as the ſector VCR increaſes, the body P draws continually nearer the centre C.

3. If the velocity be greater than falling from an infinite height, the flux. area ABFD $= \frac{+\dot{x}}{x^3}$, and the fluent $\frac{-1}{2xx}$. But at firſt the area (ſuppoſe) $= \frac{bb}{2}$, and $x = a$, an infinite line; therefore ABFD $- \frac{bb}{2} = \frac{1}{2aa} - \frac{1}{2xx}$, and ABFD $= \frac{bb}{2} + \frac{xx - aa}{2aaxx}$. Therefore $\sqrt{\text{ABFD}}$ is as $\sqrt{\frac{bbaaxx + xx - aa}{aaxx}} =$ (by ſubſtitution) $\sqrt{\frac{sxx - aa}{aaxx}}$. But in V, $x = r$, and $\sqrt{\text{ABFD}} = \sqrt{\frac{srr - aa}{aarr}} = \frac{Q}{r}$, and $Q = \sqrt{\frac{srr - aa}{a}}$, and $Z = \frac{Q}{x} = \sqrt{\frac{srr - aa}{ax}}$. Then Db

$$= \frac{\sqrt{srr - aa}}{2a\sqrt{\frac{sxx - aa}{aaxx} - \frac{srr - aa}{aaxx}}} = \sqrt{\frac{srr - aa}{s}} \times \frac{x}{2\sqrt{xx - rr}}$$

$=$ (by ſubſtitution) $\frac{Ax}{2\sqrt{xx - rr}}$, and Dc $= \frac{rr}{xx} \times$ Db $= \frac{Arr}{2x\sqrt{xx - rr}}$; therefore, $\frac{Arr\dot{x}}{2x\sqrt{xx - rr}} =$ flux. area VacD, or of VCX. And the Fl. : $\frac{Arr\dot{x}}{2x\sqrt{xx-rr}}$ = ſector VCX, which is as the angle VCX.

But in the ellipſis VRS let CH = z (Fig. 27), ſemi-conjugate = c. Then the elliptic ſector VCR = Fl. : $\frac{rc\dot{z}}{2\sqrt{rr-zz}}$, and CT = $\frac{rr}{z} = x$, and $z = \frac{rr}{x}$, and $\dot{z} = \frac{-rr\dot{x}}{xx}$. Then putting for z and $\dot{z}$ their values, and the ellip. ſector VCR = Fl. : $\frac{-crr\dot{x}}{2x\sqrt{xx-rr}} = \frac{c}{A} \times$ cir. ſector VCX; therefore when x = CT, the ellip. ſector VCR : cir. ſector VCX : : c : A; that is, becauſe the angle VCX is as the ſector VCX, the angle VCX is to the elliptic ſector VCR in a given ratio. And ſince CT or x continually increaſes as the ſector VCR increaſes, the body P goes continually farther and farther from the centre C.

[Ib. and the centripetal force becoming centrifugal] for then the curve VPQ will turn upwards; and, the law of the force being the ſame, it will be conſtructed the ſame way by the elliptic ſectors, taking the point A between C and V. And here the velocity will increaſe as it recedes from the centre C. But it can never revolve round this centre.

SECTION IX.

[Pr. 44, or in *antecedentia* with a celerity] for then the line mnC falls beyond s, sr being = rk, and sCk = 2rCk; and conſequently the point m falls without the circle.

[Ib. and with a leſs force] if the orbit moves ſlower in *antecedentia* than with twice the celerity of CP in *conſequentia*; for then the point m falls within the circle, between r and s.

The meaning of this prop. is this,---that the difference of the forces at different diſtances from the centre are reciprocally as the cubes of the diſtances (which forces are requiſite to make the body move in a quieſcent or revolving orbit, and the diſtances to be the ſame in both).

[Ib. cor. 1.] For mn repreſents the difference of the forces by which p revolves to n, or P to K, in equal times; and the verſed ſine of RK repreſents the force whereby a body moves from R to K in the circle in the ſame time.

[Ib. cor. 2. as half the *latus rectum*] for theſe forces are as bn or ar to bp or as. But $\text{ar} = \frac{\text{nr}^2}{\text{rD}} = \frac{\text{ba}^2}{\text{aD}}$. And asD : sp^2 : : aD : 2R; and $\text{asD} = \frac{\text{sp}^2 \times \text{aD}}{2\text{R}} = \frac{\text{ab}^2 \times \text{aD}}{2\text{R}}$; and $\text{as} = \frac{\text{ab}^2 \times \text{aD}}{\text{sD} \times 2\text{R}} = \frac{\text{ab}^2}{2\text{R}}$. Whence theſe forces are as $\frac{\text{ba}^2}{\text{aD}}$ to $\frac{\text{ba}^2}{2\text{R}}$, or as R to aC or CV, in Fig. 2, Pl. 18.

[Ib. cor. 3.] For let the force in the immoveable ellipſis be $\frac{FFA}{T_3}$, and in V it will be $\frac{FFT}{T^3}$ or $\frac{FF}{TT}$. And the force by which a body may revolve in a circle at the diſtance CV is $\frac{FFR}{T^3}$. And the difference of the forces in V (of revolving in the moveable and immoveable ellipſis) is $\frac{GGR - FFR}{T^3}$, and in every altitude A is $\frac{GGR - FFR}{A^3}$; and the force in the moveable ellipſis will be $\frac{FFA}{T^3} + \frac{RGG - RFF}{A^3}$.

[Ib. cor. 4.] After the ſame manner as in the two foregoing corollaries; becauſe when the velocity is given, the force ∝ curvature or reciprocally ∝ radius of curvature; therefore T : R : : $\frac{VFF}{TT}$: $\frac{RVFF}{T^3}$, which is the force wherewith a body may revolve in a circle at the diſtance CV. And, by cor. 1, FF : GG — FF : : $\frac{RVFF}{T^3}$: $\frac{RVGG - RVFF}{T^3}$, which is the difference of the forces (in the moveable and immoveable orbits) in the vertex V. And by the prop. $\frac{1}{T^3}$: $\frac{1}{A^3}$: : $\frac{RVGG - RVFF}{T^3}$: $\frac{RVGG - RVFF}{A^3}$, which is the difference of forces in every altitude A; and therefore the force in the moveable trajectory upk will be X + $\frac{RVGG - RVFF}{A^3}$.

[Ib. cor. 6.] In this cor. X = o, and RVGG—RVFF is compounded of all given quantities, and the force in Vpk will then be $\propto \frac{1}{A^3}$. This alſo appears from the prop. for the difference of the forces in P and p is as $\frac{1}{Cp^3}$, but the force in one of them, viz. at P, in the line VP, is o; and therefore the other force in Vpk is the difference of the forces, and is as $\frac{1}{Cp^3}$. Alſo this curve is the ſame with (Fig. 28) that in cor. 3, pr. 41; for let vad be a circle; and by this cor. 6, $<$ vcp $\propto$ $<$ vcz $\propto$ arc va $\propto$ area vca $\propto$ area vcb, which is the conſtruction in cor. 3, pr. 41; alſo cp = cz or cx, becauſe vz, ax, and bx are tangents, which is alſo the ſame conſtruction as in cor. 3, pr. 41.

[Pr. 45 (Fig. 29), but orbits acquire the ſame figure] Let the ſmall parts of the curve df, fh, hp, &c. be deſcribed by a body A in indefinitely ſmall given parts of time. And let another body B go from d in the ſame direction de, and with a velocity which is to the velocity of the former in the ſubduplicate ratio of the centripetal force of B to that of A; and let it arrive at s in the aforeſaid ſmall given part of time. Now ſince ef : rs : : force of A : force of B : : velocity^2 of A : vel.^{2} of B : : de^2 : dr^2 : : ſquare of the time of B's moving through de : ſquare of the time of B's moving through dr; therefore, by lem. 10, the body B will paſs through f. And ſince rs is every where as rd^2, it is evident the ſame curve dsf will be deſcribed by both bodies, and which de, fg touches in d, f. Alſo the velocity of A in f : vel. of A in d : : as perpendicular from C on de : to $\perp$ on fg : : vel. of B in f : vel. of B in d : : And vel. of A in f : velocity of B in f : : vel. A in d : vel. of B in d : : de : dr; that is, in a given ratio; that is, fg : to fk, and hn to hl, and px to pt are in a given ratio; and therefore when A is arrived at g, B would be at k; and ſince gh : ki : : gf^2 : kf^2 : : ed^2 : rd^2 : : ef : rs; or gh : ef : : ki : rs; that is, the centripetal of A to that of B at equal diſtances being in a given ratio, B will paſs through i, and conſequently through h. After the ſame manner it may be proved that the body B will paſs through the points m, p, u, y, and deſcribe the ſame curve with the body A.

Farther; if another body D move through $\beta\gamma \parallel$ to de, and with a velocity which is to the velocity of A as $C\beta$ to Cd, and be acted upon by a force which in the points β, δ, z, &c. is to the force in the points d, f, h, &c. reſpectively, as $C\beta$ to Cd, $C\delta$ to Cf, Cz to Ch, &c.; then, I ſay, the curve deſcribed by the body D, viz. $\beta\delta z\Phi\omega$, will be ſimilar to the curve dfhpy; for in the time that A would arrive at e, D would arrive at γ; and becauſe $C\gamma : Ce :: C\beta : Cd ::$ force at β : force at d $:: \gamma\delta : ef :: C\gamma - \gamma\delta$ or $C\delta : Ce - ef$ or Cf; and $C\beta : C\delta :: Cd : Cf$; $\gamma\delta$ is every where as $\gamma\beta^2$ in all the points between β and γ (by lem. 10), it is manifeſt the fig. $C\beta\delta$ is ſimilar to the fig. Cdf, and the tangent $\delta\varepsilon \parallel$ to fg; and becauſe the areas $Cgf : Cfd :: C\varepsilon\gamma : C\delta\beta$; wherefore when A comes to g, D comes to ε; but $gh : \varepsilon z :: Cf : C\delta :: Cg : C\varepsilon :: Ch : Cz$, as before; and $Cf : Ch :: C\delta : Cz$, whence the fig. $C\delta z$ is ſimilar to Cfh; and after the ſame manner it will be proved that the naſcent figures $Cz\Phi$, $C\Phi\omega$, are ſimilar to Chp, Cpy; and therefore the whole figure $\beta\delta z\Phi\omega$ is ſimilar to the figure dfhpy.

Or univerſally, if the orbits $\beta\delta\Phi$ and dfp are ſimilar, $C\beta \propto \gamma\delta \propto$ force $\times$ time^2 of deſcribing $\beta\delta \propto$ force $\times \dfrac{\beta\delta^2}{\text{velocity}^2} \propto$ force $\times$ (Fig. 30) $\dfrac{C\beta^2}{\text{velocity}^2}$. Therefore velocity$^2 \propto$ force $\times$ diſtance $C\beta$. Whence if $C\beta$ be given, velocity $\propto \sqrt{\text{force}}$; and if velocity $\propto C\beta$, force $\propto C\beta$ alſo, which agrees with what went before. If the force be given, velocity $\propto \sqrt{C\beta}$. If velocity be given, force $\propto \dfrac{1}{C\beta}$.

[Ib. exam. 1, l. 3.] here $\dfrac{T^3 - 3TTX + 3TX^2 - X^3}{A^3}$ is twice repeated in the Engliſh, which is wrong.

[Ib. By this collation of the terms] for ſince the ſame figure will be deſcribed by making the centripetal force proportional at equal diſtances (altering the velocity in the ſubduplicate ratio of the force); and ſince, when R is nearly $= T$, and x very ſmall, the centripetal force in the revolving ellipſis will be as $RGG - FFx$, and in this orbit (becauſe in both the denominator A^3 is the ſame) is as $T^3 - 3TTx$, therefore theſe forces ought to be proportional at all diſtances, viz. when $x = 0$, and any other indeterminate diſtance A or $T - x$; wherefore (in theſe two caſes) $RGG : T^3 :: RGG - FFx : T^3 - 3TTx ::$ (and by diviſion) $FFx : 3TTx :: FF : 3TT$, which is the conſtruction of the problem. After the ſame manner, in example 2, the centripetal force in the revolving ellipſis, and this new orbit, will be as $RGG - FFx$, and $T^n - nT^{n-1}x$, and to have the orbits ſimilar, putting $x = 0$, $RGG : T^n ::$ (putting $T - x$ for any indeterminate diſtance) $RGG - FFx : T^n - nT^{n-1}x ::$ (by diviſion) $FFx : nT^{n-1}x :: FF : nT^{n-1}$. After the ſame manner, in example 3, it will be $RGG : bT^m + cT^n :: (RGG - FFx : bT^m + cT^n - mbT^{m-1}x - ncT^{n-1}x) :: FFx : mbT^{m-1}x + ncT^{n-1}x$. Or $GG : bT^{m-1} + cT^{n-1} :: FF : mbT^{m-1} + ncT^{n-1}$. The quantity $\frac{FF}{AA} + \frac{RGG - RFF}{A^3}$ (in cor. 2, pr. 44) is univerſally as the centripetal force, whether the apſides move backwards or forwards; for $G : F :: VCp : VCP ::$ (p. 119) $pCn : pCk$. And the force in the revolving ellipſis is greater than in the immoveable one, when n (Fig. 2) is without the line ks; and in that caſe (F is leſs than G, and) $+ \frac{RGG - RFF}{A^3}$ will be affirmative. But the force is leſs when n falls between k and s, for then (F is greater than G, and) $+ \frac{RGG - RFF}{A^3}$ is negative, as it ought to be.

SECTION X.

[Pr. 50, it is evident from the conſtruction] Since VP touches the curve in P, and PB is ⊥ thereto, therefore B is the point of contact of the circle AD, and the wheel or generating circle, and therefore $BV = AO$. Alſo ſince WT is ⊥ to TV, therefore V is the point of contact of the generating circle (whoſe diameter is VW) and OS; whence VW = OR. The figures are ſimilar, becauſe their axes are as the radii of the ſpheres.

[Pr. 52, hence ſince in unequal —] ſuppoſe TR to be a ſmaller oſcillation, to find the velocity in I; take st : SR : : TI : TR; then velocity at R in the arc TR : velocity at R in SR : : TR : SR; and velocity at I in TR : velocity at R in TR : : (prop. 51) velocity at t in SR : velocity at R in SR : : $\sqrt{SR^2 - tR^2}$: SR. And, *ex equo*, velocity at I in TR : velocity at R in SR : : TR $\sqrt{SR^2 - tR^2}$: SR^2 : : $\frac{TR}{SR}\sqrt{SR^2 - tR^2}$: SR : : (becauſe SR : TR : : tR : IR as) $\sqrt{TR^2 - IR^2}$: SR.

[Ib. There are obtained from the times given] for in two unequal arcs two correſponding parts are deſcribed in equal times (by prop. 51); therefore the velocities in theſe points will be as the entire arcs; and therefore both the velocities and arcs will be known.

[Ib. And if the abſolute force of any globe] that is, the force at a given diſtance be called V, then the force at the diſtance CO is CO × V. But in a given time HY is as the force; whence CO × V is as HY, and therefore HY is deſcribed in a given time.

[Ib. cor. 1. For this time] for then AR = AC, and V is given: therefore, this time : time of ½ oſcillation : : $\sqrt{\frac{AC}{AC}} : \sqrt{\frac{AR}{AC}} : : 1 : \sqrt{\frac{AR}{AC}}$.

[Ib. cor 2. But in that caſe] for (in Fig. 2, Pl. 19), by cor. 1, 2, prop. 49, AS : Bv : : PS : PV : : 2CE : CB (in this caſe as) : : 2 : 1 : : (and by diviſion) AP : BV — PV; whence AP = 2BV — 2PV; but verſed ſine of ½PB = $\frac{BV - PV}{2}$, for coſine = ½PV; and ½VB — ½VP = verſed ſine.

[Ib. as M. Huygens] all this is demonſtrated in Keil's Philoſophy. See my large book of Mechanics, prop. 40, cor. 4.

[Pr. 56, let the projection] T is projected into P, and t into p.

[Ib. as alſo its poſition] The greater axis 2Tt is perpendicular to PO.

[Ib. and ſince the area POp] for let (Fig. 31) $\frac{\text{area OPp}}{\text{OP}}$ = sp, which is given (sp being ⊥ to PO); and $PB^2 : Pr^2 :: PB^2 - SP^2 : PS^2$; whence Ps is given, and thence SO, the angle POp, Op, and Pp, the point p, and angle OPp.

SECTION XI.

[Pr. 57, and about each other] conſidering either body as at reſt.

[Pr. 58, by the ſame forces there may be deſcribed, &c.] that is, by making the velocity to the former velocity as ($\sqrt{sp}$ to $\sqrt{CP}$, that is, as) $\sqrt{P + S}$ to $\sqrt{S}$, as appears by the prop.

[Pr. 60, in a ratio ſeſquiplicate] it ſhould be ſubſeſquiplicate.

[Ib. of the other ellipſis,] As $S^{\frac{1}{3}} : \overline{S + P}^{\frac{1}{3}}$. Let $S^{\frac{1}{3}} : a : e : \overline{S + P}^{\frac{1}{3}}$ be ∺. Then $S^{\frac{1}{3}} : \overline{S + P}^{\frac{1}{3}} :: (S : a^3 ::) \; e^3 : \overline{S + P}$. But (ſince $\overline{S + P} : e^3 :: a^3 : s$) e^3 is the firſt of two mean proportionals between S + P and S.

[Pr. 61.] Note, if a body were placed in the centre of gravity, and whoſe force would be ſufficient to cauſe one of the bodies to revolve around it, and to deſcribe the ſame figure, yet it would not cauſe the other body (if it were unequal) to deſcribe its figure, except in that law of centripetal force which is as the diſtance, where the periodic times are all equal. For, to preſerve the ſame motion as before, either body muſt be attracted to the body in the centre of gravity with a force which is as the other body, or as its own diſtance, which is the caſe of one body attracting another, and holds only (in the caſe of their being attracted by a third body in the centre of gravity) in the law of centripetal force beforementioned. And therefore in

[Pr. 62.] The foregoing note is to be obſerved.

[Prop. 64. This would be the caſe] For this only cauſes the bodies T, L, to revolve more ſwiftly round their centre of gravity D, but affects not the other bodies.

[*Ib.* with equal periodical times.] By the foregoing (in the three bodies S, T, L) the body S deſcribes an ellipſis round C; and, by conſidering the centre of gravity of S, T, as deſcribing an ellipſis (as before of T, L), it will appear the ſame way that the body L deſcribes an ellipſis round C; and as the centre of gravity of S, L, deſcribes an ellipſis, ſo T deſcribes an ellipſis alſo round C. Farther; from what went before, the centripetal force of S towards C is as $\overline{T + L} \times \overline{SC + CD}$, or $\overline{T + L + S} \times SC$. Alſo the centripetal force of T towards D is as $L \times \overline{TD + DL} + S \times TD$ (which latter part ariſes from the reſolution of the force ST into SD, DT, and DT acts towards D), or as $\overline{T + L + S} \times TD$. Now, by cor. 2, 8, prop. 4, the periodic times are in the ſubduplicate ratio of the radii directly, and the ſubduplicate ratio of the forces inverſely; therefore the periodic time of S round C : periodic time of T round D : : $\sqrt{\dfrac{SC}{SC \times \overline{T + L + S}}} : \sqrt{\dfrac{TD}{TD \times \overline{T + L + S}}}$; and are therefore equal; ſo that after one revolution the bodies all return to their firſt places.

After the ſame manner the point C, and a fourth body V, as alſo the bodies S, T, L, will deſcribe ellipſes round their common centre of gravity B, for any one, and the centre of gravity of the other three will deſcribe ellipſes; and the caſe is the ſame if there were more bodies. Alſo, as before, and by a like reſolution of forces, ſince the forces VS, VT, VL, are reſolved into VC, CS; VC, CT; VC, CL; the former acting in the direction of CV towards V, the latter to C, their centre of gravity, and are as the diſtances of the bodies therefrom, as before, they will therefore ſtill move round their centre of gravity C as before, but ſwifter, although all the four revolve round B. Alſo periodic time of V round B : periodic time of S round C : : $\sqrt{\frac{BV}{\overline{S+T+L}\times CB+BV}}$: $\sqrt{\frac{SC}{\overline{T+L\times S}\times SC+V\times SC}}$: : (that is, as) $\sqrt{\frac{BV}{\overline{T+L+S+V}\times BV}}$: $\sqrt{\frac{SC}{\overline{T+L+S+V}\times SC}}$; and are therefore equal; and if more bodies were added, all the bodies would perform their revolutions in equal periodic times. Q.E.D.

[Prop. 66, cor 2. Such is the force NM.] This force always acts from P in a direction parallel to TS, and from M towards T.

[Ib. cor. 6, the periodic time will be increaſed,] for then the periodic time is $\propto \frac{\text{rad.}^{\frac{3}{2}}}{\sqrt{\text{force}}}$, which is a greater ratio than rad.$^{\frac{3}{2}}$, becauſe the force is diminiſhed.

[Ib. cor. 7, the upper apſis to go backwards] for $\frac{n^2}{m^2} - 3 > -2$. And $\frac{n^2}{m^2} > 1$; therefore n is $>$ than m. Alſo put $D =$ diſtance ($r =$ a given quantity), and the force is $\left(\frac{D^3+r^3}{D^2}\right.$, or rather) as $\frac{1}{D^2+\frac{1}{D}}$, and when the diſtance increaſes, the decreaſe of the force is $D^2+\frac{1}{D}$, which is leſs than $D^2+\frac{D^3}{D}$, which is the decreaſe according to the duplicate ratio. Alſo at the conjunction, &c. the force of P towards T is the centripetal force of $T + LM - TM$, or the centripetal force of $T - KL$. Alſo the decreaſe of the force is as $d^2 - \frac{1}{d}$, which is greater than $d^2 - \frac{d^3}{d}$, or the decreaſe according to the duplicate ratio. And $\frac{n^2}{m^2} - 3 < -2$; and $n < m$.

[Ib. cor. 7. The truth of this] for then, the farther P recedes from T, the more it is attracted towards the bodies S, S, and therefore leſs towards T than it would otherwiſe be.

[Ib. cor. 8, when the apſides are in the ſyzygies] for then NM is greater and LM leſs than before.

[Ib. cor. 9. Now therefore —] For the ratio of KL to LM is leaſt when the apſides are in the quadratures, and greateſt in the ſyzygies.

[Ib. For the forces LM] as is ſhewn in cor. 7.

[Ib. cor. 10, from the ſyzygies to the quadratures] for after the ſyzygies, the body P, by the action of S, is made to move in lines which ſucceſſively cut the plane TPS at greater and greater angles. And for the ſame reaſon, at the 90° from the octants (between C and A), the body is drawn into lines (or little planes) which cut TPS at greater and greater angles, which before 90° cut it at leſs and leſs.

[Ib. and by a like reaſoning] for the inclination is increaſed from u to D (Fig. 32), diminiſhed from D to S, increaſed from S to C, diminiſhed from C to u; as is plain by ſuppoſing T and any point p, wherein the body is, to be joined; and the plane pTt to revolve round pT, till it paſs through the new place of the body p (which it acquires either above or below the plane pTt) by the (action of S upon it, or the) force NM; and then it will appear how that new acquired plane cuts the plane SCT, whether in a greater or leſſer angle.

[Ib. cor. 11 — from the former plane CD] Since the inclination of the orbit is diminiſhed from C to A, the interſection with TSE will move from C to B, and from D to A; and ſeeing that inclination is increaſed from A to D, the interſection (or nodes) will ſtill move towards B and towards A.

[Ib. being always either retrograde] that is, conſidering a whole revolution; for in ſome points they go forward.

[Ib. cor. 13. And ſince the cauſes and proportions] to one another, and to the force of S, &c.

[Ib. cor. 14. But ſince the forces] For SK : LM : : accelerating force of T towards S : perturbating force of S. And therefore the perturbating force of S =

$\frac{LM \times \text{accelerating force of T towards S}}{SK}$. But ſince SK = ST nearly, and LM (in its mean quantity) = PT, and the accelerating force of T towards S is as the body S directly, and ST^2 reciprocally, therefore the perturbating forces LM, NM, $\propto \frac{PT \times \text{body S}}{ST^3}$, that is, $\propto \frac{PT \times \text{bodies}}{\square\ \text{periodic time of T round S}}$. Or $\propto$ PT × denſity of S × cub. apparent diameter of S (for the apparent diameter is as the real diameter directly, and the diſtance reciprocally).

[Ib. cor. 16, ſquare of the periodical time of the body P conjunctly.] This holds as true in the ſynodic time of the body P (ſince the action of the forces that cauſe theſe errors begin and end at the quadratures, which comes to the ſame as if they begun and ended at the ſyzygies): ſee what follows.

[Ib. and hence the angular errors] theſe in one revolution are as the linear errors directly, and the radius or diſtance reciprocally; that is, as the forces and ſquare of the time of revolution of P directly, and the diſtance TP reciprocally; that is (becauſe the force $\propto$ TP), as ſquare of the time of revolution.

[Ib. Let theſe ratios] The angular errors of P obſerved from T are as $\frac{\text{forces} \times \text{time}^2}{TP}$, that is (by cor. 14), as $\frac{PT \times \text{body S} \times \text{time}^2}{PT \times \square\ \text{T's periodic time}}$, that is, in the time of one revolution of P (and if S be given), as $\frac{\square\ \text{time of P's revolution}}{\square\ \text{periodic time of T}}$.

[Ib. both theſe motions will be as the periodical time of the body P directly,] for the angular motion or velocity, or the mean angular errors, are as the ſum of all the angular errors in any time directly, and the time reciprocally; that is (in the periodic time), as $\frac{\text{body S} \times \square\ \text{time of P's revolution}}{\text{time of P's revolution} \times \square\ \text{periodic time of T}}$, that is, as body S × time of P's revolution directly, and the ſquare of the periodic time of T reciprocally.

[Cor. 20. And thence the greateſt height of the water. This is explained by Worſter in his (Experimental) Philoſophy, page 72. Alſo ſee my Geography, prop. 12, ſect. 1.

SECTION XII.

[Prop. 79. As the lineola Dd,] when the radius of the ſphere is given; and when the radius is not given, as Dd and the radius conjunctly; therefore AS, and alſo PS or PE, may be of any magnitude.

[Ib. the ſum of all the rectangles PD × Dd] that is, as $\frac{PF + PD}{2} \times DF$, or $\frac{PF + PD}{2} \times \overline{PF - PD}$, or $\frac{PF^2 - PD^2}{2}$.

[Prop. 80. The whole force of the ſphere will be as the whole area ANB.] Alſo it is manifeſt that the attraction or force of any part of the ſphere FEB is as the correſpondent part of the area DNB.

[Pr. 81, ex. 1.] The area SL into AB $= SL \times AB$, becauſe both SL and AB are given. And the area LD into AB is a trapezoid, whoſe baſe is AB, and the two parallel ſides LA and LB; for as the point D removes nearer A, the ordinate LD decreaſes (from B, where it is LB) to A (where it is LA); and that area is $\overline{\frac{LA + LB}{2}} \times AB$, or $\frac{\overline{LA + LB}}{2} \times \overline{LB - LA}$, that is, $\frac{LB^2 - LA^2}{2}$; or it is $\frac{2LA + AB}{2} \times AB$, that is, $LS \times AB$. And the ordinate $\frac{ALB}{LD}$ is the ordinate of an hyperbola between the aſymptotes whoſe centre is L, and one aſymptote LB; for ALB is a given rectangle, and the nature of the hyperbola is that $\frac{ALB}{LD}$ (LD the part of the aſymptote being taken as the axis) is as the ordinate erected on D: whence the reaſon of the conſtruction will be evident; for aABb is the aforeſaid trapezoid, by conſtruction; alſo in the hyperbola ab the rectangle $LB \times Bb$ is given, but that (by conſtruction) is = the rectangle ALB, when the hyperbolic area aABb is that deſcribed by the ordinate $\frac{ALB}{LD}$. And the difference of that and the trapezoid (which is the difference of the two firſt areas) is the area aba required.

[Ib. Ex. 2, the third $\frac{ALB \times SI}{2LD^2}$) this is eaſily calculated by fluxions. LD is a flowing quantity; the fluxion of the area is $\frac{ALB \times SI}{2} \times LD^{-2} \times \dot{LD}$; and the whole area is $\frac{ALB \times SI}{-2} \times LD^{-1}$, that is, $\frac{ALB \times SI}{-2LD}$, or (when LD $=$ LB) $\frac{ALB \times SI}{-2LB}$: from this take the area (when LD $=$ LA, viz.) $\frac{ALB \times SI}{-2LA}$, and the difference is the area ſought, $\frac{ALB \times SI}{2LA} - \frac{ALB \times SI}{2LB}$.

[Ib. Ex. 3.] The fluxion of the firſt area is $\frac{SI^2 \times SL}{\sqrt{2SI}} \times LD^{-\frac{2}{3}} \times \dot{LD}$, and the whole area is $\frac{2SI^2 \times SL \times LD^{-\frac{1}{2}}}{-\sqrt{2SI}}$ or $\frac{2SI^2 \times SL}{-\sqrt{2SI} \times \sqrt{LB}}$, and that part of the area on AB is $\frac{2SI^2 \times SL}{\sqrt{2SI}} \times \overline{\frac{1}{\sqrt{LA}} - \frac{1}{\sqrt{LB}}}$. The fluxion of the ſecond area is $\frac{SI^2}{2\sqrt{2SI}} \times LD^{-\frac{1}{2}} \times \dot{LD}$; and the whole area $\frac{SI^2}{\sqrt{2SI}} \times LD^{\frac{1}{2}}$; and the area on AB is $\frac{SI^2}{\sqrt{2SI}} \times \overline{\sqrt{LB} - \sqrt{LA}}$. Alſo the fluxion of the third area is $\frac{SI^2 \times ALB}{2\sqrt{2SI}} \times LD^{-\frac{5}{2}} \times \dot{LD}$; and the whole area $\frac{SI^2 \times ALB}{-3\sqrt{2SI}} \times LD^{-\frac{3}{2}}$; and the area on AB is $\frac{SI^2 \times ALB}{3\sqrt{2SI}} \times \overline{\frac{1}{\sqrt{LA^3}} - \frac{1}{\sqrt{LB^3}}}$.

[Ib. And thoſe after due reduction] LA, LI, LB, are $\div$; for SP $= \frac{SA^2}{SI}$, PI $= \frac{SA^2}{SI} -$ SI, LI $= \frac{SA^2 - SI^2}{2SI}$. And

$LA = PS - AS - LI = \frac{SA^2 + SI^2}{2SI} - SA$. And $LB = \frac{SA^2 + SI^2}{2SI} + SA$. But $\frac{SA^2 + SI^2}{2SI} - SA : \frac{SA^2 - SI^2}{2SI}$ $\frac{SA^2 + SI^2}{2SI} + SA$, are ∺, as will appear by multiplying them. Hence $\sqrt{LB} - \sqrt{LA} = \sqrt{2SI}$. For $2\sqrt{\overline{LB \times LA}} = 2SI = 2LS - 2SI = LB + LA - 2SI$. And $LB + LA - 2\sqrt{\overline{LB \times LA}} = 2SI$. And (by evolution) $\sqrt{LB} - \sqrt{LA} = \sqrt{2SI}$. Therefore the firſt area $\left(= \frac{2SI^2 \times SL}{\sqrt{2SI}} \times \frac{\sqrt{LB} - \sqrt{LA}}{\sqrt{\overline{LB \times LA}}}\right) = \frac{2SI^2 \times SL}{LI}$. And the ſecond area $= SI^2$. And $\sqrt{LB^3} - \sqrt{LA^3}$ $(= \overline{LB + LA} \times \overline{\sqrt{LB} - \sqrt{LA}}$ $\overline{+ \sqrt{LB} \times \sqrt{ALB} - \sqrt{AL} \times \sqrt{ALB}} = 2LS \times \sqrt{2SI} + \sqrt{ALB} \times \sqrt{2SI} = \overline{2LS + LI} \times \sqrt{2SI}) = \overline{3LI + 2SI} \times \sqrt{2SI}$. Whence the third area $= \frac{SI^2 \times ALB}{3\sqrt{2SI}} \times \frac{\sqrt{LB^3} - \sqrt{LA^3}}{\sqrt{ALB^3}} = \frac{SI^2 \times \overline{3CI + 2SI}}{3\sqrt{ALB} = 3LI} = SI^2 + \frac{2SI^3}{3LI}$.

[Pr. 83, let us ſuppoſe] it ſhould be, let that ſuperficies be not a purely, &c. as it is in the original.

SECTION XIII.

[Pr. 90, cor. 1, 2] for the fluxion of the area is as $D^{-n}\dot{D}$, and the area as $\frac{D^{1-n}}{1-n}$, or as $\frac{1}{D^{n-1}}$; and the area ALIH as $\frac{1}{PA^{n-1}} - \frac{1}{PH^{n-1}}$.

[Pr. 91, cor. 1. And the other part $\frac{PF}{PR}$] for the fluxion of that area is $= \frac{PF}{PR} \times \dot{PF} = \frac{PF}{\sqrt{RF^2 + PF^2}} \times \dot{PF} = \overline{RF^2 - PF^2}^{\frac{1}{2}} \times PF \times \dot{PF}$. And the area $= \overline{RF^2 + PF^2}^{\frac{1}{2}} = PR$. And (the area on PB is PE — AD; and on PA, PD — AD. And their difference) PE — PD is the area on AB.

[Pr. 91, cor. 2.] This is calculated in the Appendix, by the help of the quadratures of curves, which ſee in Harris's Lex., vol. 2.

And the curve KRM is a conic ſection; for the indetermined quantities riſe only to two dimenſions. Let SA $= r$, SC $= c$, BE $= u$, PB $= b$, $ER^2 = PD^2 = PE^2 + ED^2 = \overline{b - u}^2 + \frac{cc}{rr} \times \overline{2ru - uu} = bb + \overline{\frac{2cc}{r} - 2b} \times u + \overline{1 - \frac{cc}{rr}} \times uu$.

[Sch. Pr. 93, and I ſuppoſe the force proportional.] For (Fig. 33) conſider A, B, as flowing quantities, and let $\dot{A}$ be given $= o$. Now ſince $A^{\frac{m}{n}} = B$; therefore when A becomes $A + \dot{A} = A + o$, B becomes $B + \dot{B}$, and then $B + \dot{B} = \overline{A + o}^{\frac{m}{n}} = A^{\frac{m}{n}} + \frac{m}{n} oA^{\frac{m-n}{n}} + \frac{m^2 - mn}{2n^2} ooA^{\frac{m-2n}{n}} + \&c.$ Now if any quantity $\overline{A + o}^{\frac{m}{n}}$ (where o is infinitely ſmall) be involved, the firſt, ſecond, third, &c. term will be reſpectively as the flowing quantity ($A^{\frac{m}{n}}$), the firſt, ſecond, &c. fluxion of that quantity, as is demonſtrated in the quadrature of curves, ſch. to pr. 11 (for which ſee Harris's Lexicon). Therefore $\frac{m^2 - mn}{2n^2} ooA^{\frac{m-2n}{n}}$ is as the ſecond fluxion of $A^{\frac{m}{n}}$. Now if a (centrifugal) force act from the line CD in the direction B, and a body move in the curve CZ, the fluxion of A will continue the ſame always, and the force acting upon the body will be every where as the ſecond fluxion of B, or of $A^{\frac{m}{n}}$ (its equal), and therefore as $\frac{m^2 - mn}{2n^2} ooA^{\frac{m-2n}{n}}$, or (becauſe o is given) as $\frac{m^2 - mn}{n2} A^{\frac{m-2n}{n}}$, or as $\frac{m^2 - mn}{n^2} B^{\frac{m-2n}{m}}$. Or $\frac{mn - m^2}{n^2} B^{\frac{m-2n}{m}}$ is as the centripetal force. This may be more eaſily ſhewn by fluxions only, by finding the ſecond fluxion of (B or) $A^{\frac{m}{n}}$.

SECTION XIV.

[Pr. 94, equal to the ſquare of HM] (Fig. 34) for imagine a diameter drawn through H (Fig. 35), and an ordinate to it through I, the abſciſſa will be = MI, and the ordinate = MH. Alſo, completing the figure, the tangent IP cuts from the diameter Ha, Hp = Hu = MI; therefore the triangles HLP, MLI, are equal and ſimilar, and HL = LM. And the *latus rectum* to Ha is always the ſame, whatever the angle of incidence be, if the velocity be given; for the line Hd that the body will deſcribe in a given time is given; and the line dg that the body approaches the plane in a given time is given; and therefore $\frac{Hd^2}{Hz}$, or $\frac{\overline{zg}^2}{zH}$, or the *latus rectum*, is given.

[Pr. 96, ſch. of the ſecants] of the angles between the line of incidence and the plane.

[Pr. 97, cor. 2] The curve lines CP, CQ (being every where perpendicular to AP, DK), are compoſed of arcs of circles; therefore PD is the increment of AP or AC, and QD the decrement of QK or CK; and therefore theſe increments are as the ſines of incidence and emergence. And, *è contra*, if PD, QD, are as the ſines of incidence and emergence, a body moving in the line PD ſhall emerge in the line QDK, by this prop.

[Pr. 98, and therefore QS to be always equal to CE;] (Fig. 36). For ſince qqqq and ssss are always perpendicular to qs, therefore ſuppoſing QS and 1qs to interſect in K, then KQ — KS = K. 1q — KS, that is, qs = 1q.s. Alſo let 1q.K and 2q.s interſect in T; then 1q.s = 1q.T — ST = 2T — sT = 2q.s. Alſo 2q.s = 2q.V — SV = 3q.V — SV = 3q.S; and ſo on, till at laſt qs, coinciding with CE, will be equal thereto. Therefore QS is every where equal to CE.

[Ib. ſch.] See the Author's Optics, where all theſe things are ſhewn.

BOOK II.

SECTION I.

[Pr. 2, cor. For if that area] by Marq. Hoſpital's Conic Sect., art. 219; and my Conic Sect., prop. 86, hyperbola, cor. 6.

[Ib. of the right line AC] by the converſe of lem. 1; but the contrary does not hold, that is, if any area ABGD be taken for the time, that DC ſhall repreſent the velocity, or AD the ſpace.

[Pr. 3. And the reſiſtance] The force of gravity may be compared with the reſiſtance of the medium; for they may both be conſidered as uniform preſſure.

[Ib. or AC to ½AK] For ſuppoſe a tangent to be drawn to the point B, then Kq : (ſubtangent =) AC : : kq : Bk or AK. And in general in any area, as MstN, it will be, AC : ½MN + NC : : (by the nature of the hyperbola) $\frac{Ms + Nt}{2}$: AB or Mm. And by diviſion AC : AM + ½MN : : $\frac{Ms + Nt}{2}$: $\frac{ms + nt}{2}$: : area sMNt : area smnt. But AG : AM + ½MN : : force of gravity : reſiſtance in the middle of the fourth time. Therefore sMNt : smnt : : force of gravity : reſiſtance in the midſt of the fourth time, and ſo of the reſt.

And the like demonſtration holds (Fig. 37) in aſcending motion; for let the rectangle ABDG be divided into innumerable rectangles Dk, Kl, Lm, Mn, &c. which ſhall be as the decrements of the velocities produced in ſo many equal times; then will AE, Ak, Al, Am, &c. be as the whole velocities, and therefore as the reſiſtances of the medium in the beginning of each of the equal times. Let AC : AK : : force of gravity : reſiſtance in the beginning of the ſecond time; then to the force of gravity add the reſiſtances, and DEHC, KkHC, LlHC, MmHC, &c. will be as the abſolute forces, or ∝ decrements of the velocities, ∝ Dk, Kl, Lm, Mn, &c. and therefore ÷÷; whence DGqK = KqrL = LrsM = MstN, &c. will be ∝ the equal times or forces. But AC :

MC : : Ms : AB. And by diviſion AC : AM : : Ms : ms : : area sMNt : area smnt : : (and therefore as) force of gravity : reſiſtance in the fourth time; and ſo of the other areas. Therefore ſince DGqK, KqrL, LrsM, MstN, are ∝ the gravitating forces, the areas GEkq, qklr, rlms, msnt, ∝ reſiſtances in each time, ∝ velocities ∝ ſpaces deſcribed. And, by compoſition, in the times DGrL, DGBA, the ſpaces deſcribed will be as GElr, GEB. Q.E.D.

[Ib. cor. 3.] For the diff. ſpaces ∝ velocities (that is, by cor. 2) ∝ CA, CK, CL (∺), or ∝ AK, KL, LM (÷), by lem. 1 (Fig. 3).

[Ib. let that alſo be diſtinguiſhed] This is true in this law of reſiſtance; becauſe the motion loſt is ∝ velocity ∝ remaining velocity ∝ ſpaces deſcribed.

[Pr. 4, which is the locus of the point r. Q.E.D.] all the reſt is plain by prop. 2 and 3, and cor. pr. 2.

Ib. cor. 1, that is, if the parallelogram] For DA : CP : : DR : RX $= \frac{DR \times CP}{DA}$. But DAB $= \overline{DC - AC} \times AB$ $=$ (by the nature of the hyperbola) $\overline{AB - AQ} \times DC = QB \times DC =$ (by conſtruction) $N \times CP$. And DA $= \frac{N \times CP}{AB}$. Wherefore RX $= \frac{DR \times CP \times AB}{N \times CP} = \frac{DR \times AB}{N}$.

[Ib. cor. 4, DraF is alſo given.] by this prop.

[Ib. cor. 6.] for then 2DP $\propto \frac{\text{lat. rect.}}{\text{reſiſtance}} \propto \frac{\text{lat.}}{\text{velocity}} \propto \frac{\text{velocity}^2}{\text{velocity}} \propto$ velocity.

[Ib. cor. 7, the ratio $\frac{Ff}{DF}$] For when this ratio is the ſame with the other, the curves deſcribed (and ſuppoſed to be deſcribed) in theſe two caſes will be ſimilar.

SECTION II.

Pr. 5 had been better expreſſed thus (though the tranſlation agrees with the original).

If a body is reſiſted in the duplicate ratio of its velocity, and moves by its innate force only through a ſimilar medium, *and the ſpaces be taken equal, I ſay, that the times of their deſcription are in a geometrical progreſſion increaſing ; and that the velocities at the beginning of each of the times are in the ſame geometrical progreſſion (decreaſing or) inverſely.*

For it is plain that the times may be taken in ſuch a geometrical progreſſion as that the ſpaces cannot be equal, nor the velocities in the ſame inverſe progreſſion. But if the ſpaces are equal, the times and velocities will be in a geometrical progreſſion, inverſe to one another. The demonſtration is the ſame, however the propoſition be expreſſed ; for it being proved that the lines AB, Kk, Ll, &c. being ſquared, the ſquares are as the differences of the lines; and alſo that the ſquares of the velocities are as the differences of the velocities. Therefore if AB and Kk be taken as the velocities in the beginning of the times AK, KL, and an hyperbola be drawn through the points B, k, to the aſymptote CD (whoſe centre let be C), all the other velocities will be as the lines Ll, Mm, &c. becauſe the progreſſion, of the lines as well as the velocities continues the ſame all along. Whence the ſpaces (in theſe equal times) will be as the areas Ak, Kl, Lm, &c. and in the time AM the ſpace will be as AMmB. Now conceive the area AMmB to be divided into the equal areas Ak, Kl, Lm, &c. Then will CA, CK, CL, &c. be ∺ increaſing ; and the parts (by lem. 1) AK, KL, LM, &c. which are as the times, will be in the ſame progreſſion, increaſing. Alſo the velocities AB, Kk, Ll, &c. (which are reciprocally as CA, CK, CL, &c.) will be in the ſame progreſſion inverſely. Q.E.D.

[Pr. 7, and times conjunctly] very ſmall particles of time.

[Ib. they will always deſcribe ſpaces,] for the parts of ſpace deſcribed in thoſe ſeveral particles of time are as the reſpective velocities × thoſe ſeveral parts of time; that is, as the firſt velocity × each part of time. And, by compoſition, the whole ſpaces are as the firſt velocities × whole times. Q.E.D.

[Pr. 7, cor. 3.] dele, Let thoſe diameters applied to that power.

[Pr. 8.] in Fig. 1, Pl. 2, k ought to be beyond l in reſpect of A, &c. becauſe the reſiſtance decreaſes. But the demonſtration is the ſame any way. But this figure is not in the 1ſt or 2d edit. of the original. In prop. 8 and 9, AC repreſents the relative gravity, or its weight in the fluid.

[Pr. 9, cor. 7.] For the greateſt velocity is given (by pr. 8, cor. 2, 3). And thence the time of acquiring that velocity in free ſpace. And ABNK (ABnk) being given, there is given AK, and AP (Ap), and ATD (AtD). Then ADC : ADT (ADt) : : time of acquiring that greateſt velocity in free ſpace : time ſought.

[Pr. 10, equal among themſelves] and very ſmall.

[Ib. or $\frac{MI \times NI}{HI}$] as appears by ſimilar Δs, produced by letting fall a ⊥ from N on HI.

If the velocity in GH be greater than the velocity in HI, the decrement is $\frac{GH}{T} - \frac{HI}{t}$, and ariſes from the reſiſtance and gravity together (becauſe gravity draws the body from the tangent into the arc HI); and if gravity act not, that decrement would be greater (for gravity accelerates the motion) by $\frac{2MI \times NI}{t \times HI}$; therefore the decrement by the reſiſtance alone is $\frac{GH}{T} - \frac{HI}{t} + \frac{2MI \times NI}{t \times NI}$.

[Ib. will be NI;] for Q being a given (ratio or) quantity, Qo will always repreſent MN, whatever the magnitude of o be.

And the ordinate DI $=$ CH $-$ MI; alſo the value of MI in the ordinate EK is (becauſe o becomes 2.o, and ſubſtituting $2 \times o$ for o) $2Qo + 4Ro^2 + 8Ro^3 +$ &c.; and therefore EK $= CH - 2Qo - 4Ro^2 - 8So^3 -$ &c. Alſo in the ordinate BG, when o becomes $-$ o, the value of MI (by ſubſtituting $-$ o for o) will be $- Qo + Ro^2 - So^3 +$ &c. And BG $=$ CH $-$ MI (as before) $= P + Qo - Ro^2 + So^3 -$ &c.

[Ib. or $\frac{R+\frac{3}{2}So}{R}$] for (becauſe o is infinitely ſmall) R, R + $\frac{3}{2}$So, R + 3So, are both arithmetical and geometrical proportionals.

[Ib. by ſubſtituting the values —] becomes $\overline{1+\frac{3So}{2R}} \times \overline{o\sqrt{1+QQ}-\frac{QRoo}{\sqrt{1+QQ}}} - o\sqrt{1+QQ} - \frac{QRoo}{\sqrt{1+QQ}} + \frac{\overline{2Qo+2Ro^2}\times\overline{Ro^2+So^3}}{o\sqrt{1+QQ}+\frac{QRoo}{\sqrt{1+QQ}}}$ = (neglecting the ſuperfluous powers of o, or the quantities infinitely leſs than the other) $o\sqrt{1+QQ}-\frac{QRoo}{\sqrt{1+QQ}}+\frac{3So^2}{2R}\sqrt{1+QQ}-o\sqrt{1+QQ}-\frac{QRo^2}{\sqrt{1+QQ}}+\frac{2QRoo}{\sqrt{1+QQ}}=\frac{3Soo}{2R}\sqrt{1+QQ}$.

[Ib. *latus rectum* $\frac{HN}{NI}$] or $\frac{HI^2}{NI}$ = (neglecting the ſuperfluous powers of o) $\frac{oo\times\overline{1+QQ}}{Roo}=\frac{1+QQ}{R}$.

[Ib. p. 33, that a body by aſcending from P] or any other point of the quadrant PF, in the direction of the arc of that quadrant.

[Ib. Ex. 3, the ſecond term $\frac{m}{n}o-\frac{bb}{aa}o$] or rather $\frac{bb}{aa}o-\frac{m}{n}o=Qo$; but the ſquare is the ſame either way.

[Ib. Ex. 4, for Qo] $\frac{nbb}{A^{n}+1}o-\frac{d}{e}o=Qo$.

[Ib. Sch. And therefore if a curve] for the quantity $\frac{S}{R^{\frac{4-n}{2}}}\div\left.\frac{HT}{AC}\right|^{n-1}$ is as the denſity, and if that be given the denſity is given.

[Ib. than theſe hyperbolas here deſcribed.] For let the two mediums be of the ſame denſity in the vertex of the curves; then if the bodies be projected from A, the uniform medium being more denſe, and the other leſs denſe at A, the body moving in the uniform medium will be more impeded, and conſequently will deſcend the more than that deſcribing the hyperbola, which is leſs reſiſted, and which therefore moves nearer a ſtraight line: therefore the body moving in the uniform medium is more diſtant from the aſymptotes in the vertex of the figure than the other body. But if the denſities of the mediums were ſuppoſed equal in A, then (by the ſame way of reaſoning) the body in the uniform medium would be nearer the aſymptotes in the vertex. And, generally ſpeaking, theſe curves cannot therefore differ very far.

[Ib. will touch the hyperbola in G,] for ſuppoſe the tangent TG to interſect MX. Then (by fluxions) the diſtance between that point of interſection and V is $= \frac{VX}{n}$. Then by ſim. $\triangle$s $\frac{VX}{n} : VG :: VX : TX - VG = nVG$. But $VY = nVG$. Therefore $TX = GY$.

[Ib. and the velocity] for the velocity (when the gravity is given) is as the ordinate of the parabola, that is (the abſciſſa being given, by the gravity), as the $\sqrt{\textit{latus rectum}}$.

[Ib. Rule 1.] For the velocities are as the $\sqrt{}$s of the *latus rectum* of two parabolas, that is, as $\sqrt{\frac{2XY^2}{\overline{nn+n} \times VG}}$, that is, as $\sqrt{\frac{2AH^2}{AI}}$ to $\sqrt{\frac{2Ah^2}{AI}}$, which (by ſuppoſition) are in a ratio of equality, and therefore $\frac{AH}{AI} = \frac{Ah}{Ai}$. Alſo ſince the denſities are equal, $\frac{n+2}{3XY}$ will be given, that is, $\frac{n+2}{3AH} = \frac{n+2}{3Ah}$. Whence $AH = Ah$. Whence AH, AI, remain the ſame; and HX, which is compoſed of AI and the ſubtangent to the axis XH, is the ſame alſo.

[Rule 2.] For velocity $\propto \sqrt{\frac{AH^2}{AI}} \propto$ (becauſe AH is given) $\sqrt{\frac{1}{AI}}$.

[Rule 3, and therefore AH] for let b be the ratio, then $\frac{Ah}{Ai} = b\,\frac{AH}{AI}$. And $\frac{AH^2}{AI} = \frac{Ah^2}{Ai} = b\,\frac{AH}{AI} \times Ah$. Whence $Ah = \frac{AH}{b}$; alſo, $\frac{Ah}{Ai} = \frac{AH}{bAi} = b \times \frac{AH}{AI}$. Whence $Ai = \frac{AI}{bb}$.

[Rule 4, a little greater] ſince AH decreaſes faſter than GT, therefore the ſum of all or every AH + GT is leſſer than the firſt AH + GT × number of them; and therefore the ſum of all the denſities (being reciprocal thereto) is greater than the ſum of as many denſities in A and G. But the mean denſity is = ſum of all the denſities ÷ by the number of them, and therefore is ſomething greater than the $\frac{1}{2}$ ſum of the denſities in A and G. But half ſum denſities : denſity in A : : $\frac{AH + GT}{2}$: GT. Therefore mean denſity : denſity in A : : is in a ratio a little greater than $\frac{1}{2} \times \overline{AH + GT}$: GT.

[Ib. Rule 5.] for HX = AI + ſubtangent = AI + nAI, XN being the axis.

[Ib. Rule 6, by how much] for the variation of curvature in one of theſe hyperbolas, where n is great, is leaſt in the part AG, and ſo it is in the curve of projection, where the velocity is greateſt; ſo they agree in that part. But they differ more in the part GK, where the projectile approaches to a uniform motion.

[Ib. Rule 7.] AH to 2AI ought to be 3AH to 4AI (but the original is AH to 2AI).

[Ib. Rule 8, whoſe conjugate] hyperbola ſhall paſs through the point C.

[Ib. what has been ſaid] for by ſuppoſing the index n to be negative, VG (which before was as VX^{-n}) will then be as

VX^n, and the curve AG will be a parabola; and what was demonſtrated generally of the hyperbola will hold true (for any index, and therefore) for the parabola; and all things will follow as before, ſubſtituting $-n$ for n. But the computation may be made for the parabola as for the other (Fig. 38) thus: let $VG = bbXV^n = XC^n$; then the ſubtangent $ZT = nVG$. And let $XC = a$, $ng = e$, $cC = o$; then $VG = \overline{a + o}^n = a^n + na^{n-1}o + n \times \frac{n-1}{2} a^{n-2}oo + n \times \frac{n-1}{2} \times \frac{n-2}{3} a^{n-3}o^3 = P - Qo - Roo - So^3$. Whence $Q = -na^{n-1}$. $R = -n \times \frac{n-1}{2} a^{n-2}$. $S = -n \times \frac{n-1}{2} \times \frac{n-2}{3} \times a^{n-3}$. Therefore the denſity in G is as $\left(\frac{S}{R\sqrt{1 + QQ}} = \right)\frac{n-2}{3a\sqrt{1 + n^2a^{2n-2}}}$; that is, as $\frac{1}{\sqrt{a^2 + nna^{2n}}}$, or as $\frac{1}{\sqrt{a^2 + nnee}}$; that is, reciprocally as $\sqrt{XC^2 + TZ^2}$. And the ſame may be found by fluxions, putting Q, R, S, for the firſt, ſecond, third, fluxion of VG or XC^n.

SECTION III.

[Pr. 12, and inverſely as the velocity;] for the time of deſcribing a given ſpace is reciprocally as the velocity.

[Pr. 13, caſe 1, 2, 3, the decrement or increment PQ] in a given time.

[Ib. cor. and triangle do.] therefore, *ex equo*, &c.

[Ib. ſch.] For (Fig. 39) draw the indefinite line BAP, and make BD perp. and equal to BA; and draw DF, AF, parallel to BA, BD. Let AP be the velocity, $AP^2 + 2BAP$ the reſiſtance, AB^2 the force of gravity. Draw DTP, cutting FA in T, and the time of the whole aſcent will be as the triangle DTA.

For draw DVQ cutting off PQ the moment of velocity, and DTV the moment of the triangle; then the decrement of the velocity PQ will be as the reſiſtance and gravity AP^2

$+ 2BAP + AB^2$; that is, as BP^2. But the area DTV is to the area DPQ as DT^2 to DP^2, or as DF^2 to BP^2; therefore ſince the area DPQ is as BP^2, the area DTV will be as the given quantity DF^2. Therefore the area ADT decreaſes uniformly as the time, by the ſubduction of given particles DTV; and therefore is proportional to the whole time of aſcent.

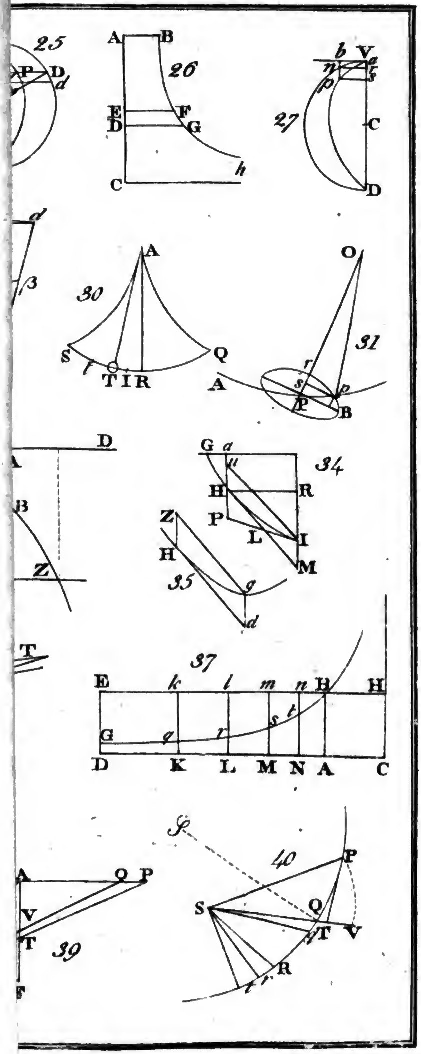
[Pr. 14. From the area DET] This ſhould be :---from the moment KLON ſubduct DTV, or $m \times BD$, the moment of DET; and is only a falſe tranſlation.

[Ib. cor. or as V^2;] for the ſpace is $\propto \square$ of the time, that is, $\propto$ the ſquare of DET, or of $BD \times M \propto$ (becauſe $M = \frac{DE \times V}{DA}$) ſquare of $\frac{BD \times DE \times V}{DA}$, that is (becauſe BD, DE, DA, are given), as V^2, or as $\frac{BD \times V^2}{AB}$.

[Ib. ſch. inſtead of the uniform reſiſtance made to an aſcending body;] this differs from gravity only in this, that it cannot generate any motion; but it acts after the ſame manner in all moving bodies in deſtroying their motion as gravity does in deſtroying the motion of aſcending bodies. Gravity acts uniformly in a given direction. The force ariſing from tenacity acts uniformly, but always in a direction contrary to the motion of the body; and, therefore, when the body is at reſt, it can induce no change in it. Now in the horizontal motion of a body in a fluid which is reſiſted in part uniformly, one may ſubſtitute the force of gravity for that uniform reſiſtance, as in pr. 8, 9, 13, 14. And in the aſcent or deſcent of the body in a fluid, inſtead of the force of gravity, one may ſubſtitute the ſum or difference of that uniform force and the force of gravity, as in theſe propoſitions.

SECTION IV.

In prop. 15. [And the decrements of theſe arcs ariſing from the reſiſtance will be as the ſquares of the times in which they are generated;] for ſince the arcs are infinitely ſmall, the reſiſtance affects the ſame as an uniform centripetal force would do.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[Ib. The decrement of the arc (Fig. 40) PQ will be] for let q, t, be the points it would arrive at, in the ſame times, in free ſpace; then $Pq = qt$, and $PQ = Qr$; therefore $Qq = qt - Qr$; and adding Qq, $2Qq = Qt - Qr = tr$; and $Qq = \frac{1}{2}tr$: but $Qq = \frac{1}{4}Rt$; therefore $tr = \frac{1}{2}Rt = Rr$, and $Qq = \frac{1}{2}Rr$.

The ſame demonſtration of this prop. holds good as well in aſcending as in deſcending motion.

[Cor 1. The velocity, &c.] for ſuppoſing PQ an arc of a circle (Fig. 41) whoſe centre is ſ; then if PQ or PT be as the velocity in both curves, TQ will be as the centripetal force in both, but in both curves $TQ = \frac{\overline{PQ}^2}{2P\text{ſ}}$; therefore $P\text{ſ} =$ rad. of the circle deſcribed with the ſame velocity.

[Cor. 4, the body will deſcend, &c.] for, by cor. 1, the velocity in P in a reſiſting medium is = velocity in a circle at diſtance SP in free ſpace; but velocity at diſtance SP in free ſpace : to velocity at diſt. $\frac{1}{2}SP$ in free ſpace (by cor. 6, prop. 4, b. I.), or prop. 34, : : as $\sqrt{\frac{1}{SP}}$: to $\sqrt{\frac{1}{\frac{1}{2}SP}}$: :, that is, $\sqrt{1} : \sqrt{2}$; therefore, &c.

[Cor. 7, or as $\frac{2}{3}AS$ to AB] (for this, ſee my Geometrical Proportion, prop. 24.)

In prop. 16. [And the reſiſtance, &c.] inſtead of this proportion in prop. 15, viz. $PQ : QR :: SQ : \sqrt{SP \times SQ}$, ſubſtitute this; $PQ : QR :: (SQ^{\frac{n}{2}} : SP^{\frac{n}{2}} ::)\ SQ : SQ + \frac{n}{2}SP - \frac{n}{2}SQ$ (by prop. 24 of my Geomet. Proportion), that is, as the deſcribing velocities, &c.

[Cor. 1.] For the reſiſtance : centripetal force : : $(\frac{1}{2}Rr =)\ \frac{\overline{1 - \frac{1}{2}n} \times VQ \times PQ}{2SQ} : (TQ =)\ \frac{\frac{1}{2}PQ^2}{Sp} :: \overline{1 - \frac{1}{2}n}\,VQ : PQ :: \overline{1 - \frac{1}{2}n}\,OS : OP$.

SECTION V.

In prop. 19. [Caſe 3, different ſpherical parts have equal preſſures,] that is, different ſpherical parts of equal magnitude.

In prop. 20. [And by a like reaſoning, &c.] Suppoſing the thickneſs of the orbs to be reciprocally as the force of gravity or the denſity of the fluid, or in the complicate ratio of them, and it will appear (as before) that the bottom is preſſed with a cylinder of the ſame fluid whoſe baſe is = the bottom, and height that of the fluid, by the ſame reaſoning as before, and prop. 19.

In ſchol. pr. 22. [By a like reaſoning, &c.] Let the force of gravity be reciprocally as the nth power of the diſtance. The ſpecific gravities at A, B, C, &c. will $\propto \frac{AH}{SA^n}, \frac{BI}{SB^n}, \frac{CK}{SC^n}$, &c. and the denſities $\propto$ ſums of the preſſures $\propto \frac{AH \times AB}{SA^n}, \frac{BI \times BC}{SB^n}, \frac{CK \times CD}{SC^n}$, &c. $\propto \frac{AH}{SA^{n-1}}, \frac{BI}{SB^{n-1}}, \frac{CK}{SC^{n-1}}$, &c. And tu, uw, &c. $\propto$ differences of the denſities $\propto \frac{AH}{SA^{n-1}}, \frac{BI}{SB^{n-1}}$, &c. And tu × th, uw × ui $\propto \frac{AH \times th}{SA^{n-1}}, \frac{BI \times ui}{SB^{n-1}} \propto \frac{1}{SA^{n-1}}, \frac{1}{SB^{n-1}}, \propto \frac{1}{SA^{n-1}} - \frac{1}{SB^{n-1}}, \frac{1}{SB^{n-1}} - \frac{1}{SC^{n-1}}$, &c.; and therefore, univerſally, $\frac{1}{SA^{n-1}} - \frac{1}{SE^{n-1}} \propto$ hyperbolic area thmy. Therefore if $\frac{SA^n}{SA^{n-1}}, \frac{SA^n}{SB^{n-1}}, \frac{SA^n}{SC^{n-1}}$, or $\frac{1}{SA^{n-1}}, \frac{1}{SB^{n-1}}, \frac{1}{SC^{n-1}}$, be ∺, then $\frac{1}{SA^{n-1}} - \frac{1}{SB^{n-1}} = \frac{1}{SB^{n-1}} - \frac{1}{SC^{n-1}}$, and the areas proportional thereto will be equal, viz. thiu = uikw; and therefore St, Su, Sw, that is, AH, BI, CK ∺. Q.E.D. Then putting n ſucceſſively = 3, 4, 0 — 1, &c. the truth of the ſaid ſcholium will appear. And ſo on *ad infinitum*.

[Other laws of condenſation, &c.] Suppoſe the compreſſing force $\propto$ rth power of the denſity; then the ſpecific gravities in A, B, &c. $\propto \frac{AH}{SA^2}, \frac{BI}{SB^2}$, &c.; and the preſſures in A, B, &c. $\propto \frac{AH}{SA} + \frac{BI}{SB} + \frac{CK}{SC} + \frac{DL}{SD}$, &c. $\frac{BI}{SB} + \frac{CK}{SC} + \frac{DL}{SD}$,

&c. Now if theſe ſums were ∺, their differences $\frac{AH}{SA}$, $\frac{BI}{SB}$, &c. would be in the ſame ∺, and then the preſſures in A, B, &c. $\propto \frac{AH}{SA}, \frac{BI}{SB}$, &c. $\propto \overline{\text{denſities}}^{r} \propto AH^{r}, BI^{r}$, &c. And therefore $AH^{r-1}, BI^{r-1} \propto \frac{1}{SA}, \frac{1}{SB}$; and AH, BI $\propto \frac{1}{\overline{SA}^{\frac{1}{r-1}}}, \frac{1}{\overline{SB}^{\frac{1}{r-1}}}$. Then putting $r = \frac{4}{3}, \frac{5}{3}, 2$, &c. all things will appear as in this ſchol. But now the aforeſaid ſums are not in geometrical progreſſion, becauſe it is plain their differences are not, viz. $\frac{AH}{SA}$, $\frac{BI}{SB}$, &c.; for if $\frac{AH}{SA}$, $\frac{BI}{SB}$, &c. ∺, and ſeeing SA, SB, &c. are ∺, therefore (by pr. 18, cor. 2, of my Geom. Proportion) AH, BI, &c. would be ∺, which is falſe (except in the caſe of prop. 21); therefore neither theſe differences nor ſums are ∺. And conſequently the concluſions of this ſchol. in this reſpect are not true; and therefore it muſt be preſumed that the author meant them only to be nearly, and not perfectly, true.

In Schol. prop. 23. [All theſe things are, &c.] It ſeems to me that the prop. holds true, though the centrifugal force be extended to all diſtances in any given law, as well as when it ends at the next particle in the ſame given law. For, ſuppoſing the particles of the fluid to be placed in parallel planes, and that theſe planes act on each other, the force of compreſſion on any one plane will be made up of the ſeveral forces of all the other equidiſtant parallel planes. Therefore ſince the diſtances of all theſe planes before compreſſion are reſpectively proportional to their diſtances after compreſſion from this given plane, their actions on the given plane before and after compreſſion will alſo be proportional (ſeeing their actions or forces are as ſome power of the diſtances); and their ſums will be alſo proportional to thoſe of any two correſponding planes, viz. as the neareſt to the neareſt. In theſe ſort of particles there will be required a greater force to produce an equal condenſation of an equal quantity of the fluid;

but the denſity will be proportional to the compreſſion, as before.

SECTION VI.

In prop. 24. Cor. 5. —— for let m, M = quantities of matter, w, W = weights, t, T = times, for the common pendulum l. Alſo L = new pendulum, T = new time, for M, and W. Then m : M : : wtt : WTT (by the prop.) : : $\frac{wtt}{t} : \left(\frac{wtt}{t} = \right) \frac{WTT}{L}$. And if m, M; t, T, are equal, w $\propto$ l, which is cor. 4.

In prop. 27. [If the reſiſtance in the arc B, &c.] If the reſiſtance in the arc A were to the reſiſtance in the arc B as AA to AB, the times would be equal; and therefore reſiſtance AA in the arc A cauſes the exceſs of time above that in a non-reſiſting medium; and reſiſtance AB in the arc B cauſes the exceſs of time in B above that in a non-reſiſting medium, equal to the former exceſs (becauſe with theſe reſiſtances), they are deſcribed in equal time. But now if B be deſcribed with a reſiſtance BB, the exceſs of time will be cauſed by that reſiſtance.

[But thoſe exceſſes, &c.] Now the exceſſes of time cauſed by deſcribing B with the reſiſtances AB, BB, reſpectively are as the ſpaces left undeſcribed after the given time; that is, as the reſiſtances AB, BB, nearly, or as A to B, viz. as the arcs A, B. And ſince the exceſs of time by deſcribing A with reſiſtance AA = exceſs of time by deſcribing B with reſiſtance AB, therefore it follows that exceſs of time in A : to exceſs of time in B : : as A : to B nearly.

In prop. 29. [Area PIEQ may be to the hyperbolic area PITS : : as BC to Ca; and that the area IEF may be to the area ILT as OQ to OS.]

That this is poſſible (aſſuming P at pleaſure) may be thus ſhewn. Let the areas PIEQ, PITS, be taken (as BC to Ca) indefinitely ſmall; then PIEQ muſt be greater or at leaſt equal to PITS: ſuppoſe them equal; then will PIEQ > PILS, and IF > IL (and more ſo, if PIEQ > PITS); but when S, P, Q, coincide, OS to OQ is a ratio of equality; therefore in this caſe IFE to ILT (or IF to IL) $>$ OQ to OS (or however not leſs) increaſe the areas PIEQ and PITS in the ſame ratio, and the ratio of OQ to OS will alſo increaſe and converge at length to the ratio of IEF to ILT; which ſhews the poſſibility of what was required (but as to an actual ſolution I ſhall refer it to an algebraic proceſs). This being done as required, then the points O, S, P, Q, are all fixed, and the point R only variable.

[And the increment, &c.] for the fluxion of $\frac{IEF}{OQ}$ OR — IGH to the fluxion of — PIGR : : as $\frac{IEF}{OQ}$ — HG to — RG : : or as HG — $\frac{IEF}{OQ}$ to RG. Or rather thus: the increment of $\frac{IEF}{OQ}$ OR — IGH is as $\frac{IET}{OQ}$ — HG × — Rr = HG × Rr — $\frac{IEF}{OQ}$ × Rr. And the increment of PIGR is as RG × — Rr; and therefore the decrement (becauſe the decrement is a negative increment) of PIGR is as RG × Rr.

In cor. to pr. 30. [Now if the reſiſtance DK be in the duplicate, &c.] for the velocities being as the ordinates of a circle or ellipſis deſcribed on aB, the reſiſtances (being as the ſquares of theſe velocities) are as the ſquares of theſe ordinates, that is, as the rectangles of the abſciſſas. Therefore the reſiſtances DK, OV, are reſpectively as aDB (aOB =), OB^2. But this is the property of the parabola aVB deſcribed to the axis VO, by cor. 2 to pr. 4 of the parabola.

Gen. ſch. pr. 31, p. 75. [But in leſſer oſcillations ſomewhat greater, &c.] that is, the difference in the leſſer arc to the difference in the greater arc is in a greater ratio than the ſquares of theſe leſſer and greater arcs.

[P. 75.] for $\frac{1}{2}$ the chords of theſe arcs, read the chords of thoſe half arcs.

[Ib. p. 76, 77. As $\frac{7}{11}$ AV + $\frac{7}{10}$ $BV^{\frac{3}{2}}$ + $\frac{3}{4}$ CV^2.] Theſe three members are taken to denote three laws of reſiſtance, viz. the firſt in the ſimple ratio of the velocity, the third in the duplicate ratio thereof, and the ſecond partly in the ſimple, partly in the duplicate ratio (or in a mean between them); and $\frac{7}{10}$ the coefficient thereof is a middle one between $\frac{7}{11}$ and $\frac{3}{4}$.

[As 0,041748 to 121] it ſhould be, as 0,041778 to 121.

[P. 78. It is manifeſt that the force, &c.] for the forces of reſiſtance and gravity are as their effects; to wit, as the velocity loſt (by reſiſtance) and velocity gained (by gravity) in the ſame time as 1 to $376\frac{1}{50}$.

[I alſo counted—I leave the calculation, &c.] The calculation after the preceding manner will be as follows:

Mean oſcillations - - - - - -	$3\frac{1}{2}$	7	14	28	56	112
Difference between firſt deſcent and laſt aſcent	$\frac{1}{2}$	1	2	4	8	16
Difference of arcs deſcribed in the deſcent and ſubſequent aſcent - - -	$\frac{1}{748}$	$\frac{1}{272}$	$\frac{4}{325}$	$\frac{12}{250}$	$\frac{24}{125}$	$\frac{48}{68}$

Alſo theſe in the greater arcs are nearly in the duplicate ratio of the velocities, but in the leſſer arcs ſomewhat greater (theſe arcs are in the laſt ſeries of the table).

Let V = velocity in ſecond, fourth, and ſixth caſes = 1, 4, 16, reſpectively; and the difference of the arcs will be, in the

2d Caſe $= \frac{1}{272} = A + B + C$.
4th Caſe $= \frac{12}{250} = \frac{6}{125} = 4A + 8B + 16C$.
6th Caſe $= \frac{48}{68} = \frac{12}{17} = 16A + 64B + 256C$.

Whence A = ,0005098; B = ,0005882; C = ,0025784. Therefore the difference of the arcs is as $,0005098V + ,0005882V^{\frac{3}{2}} + ,0025784V^2$. And conſequently the reſiſtance of the globe in the middle of the arc will be to its weight (by cor. pr. 30) as $,00032442V + ,00041174V^{\frac{3}{2}} + ,0019338V^2$ to 121. Therefore the reſiſtance will be to the weight, in caſe 2d, 4th, and 6th, as ,00267 to 121; ,0355324 to 121; ,5265949 to 121, reſpectively.

Note—You may take any other numbers (that are in the ſame proportion with theſe above) for V; for example, I took $\frac{1}{4}$, 1, and 4 = V, in the 2d, 4th, and 6th caſes, and A will be = ,002039; B = ,004706; C = ,041255; and the reſiſtance to the weight (at the laſt) comes out the ſame as before.

In the 6th caſe, the point marked in the thread deſcribed an arc of 112 — $\frac{48}{68}$ inches = 111$\frac{5}{17}$; therefore the centre of the globe deſcribes an arc = 115$\frac{167}{187}$ = 115$\frac{8}{9}$ nearly; and its velocity is (nearly) the ſame as in deſcending an arc 57$\frac{17}{18}$ of a cycloid (whoſe ſemi-arc is 126), or = velocity acquired by falling perpendicularly through the verſed ſine (or abſciſſa) of that arc 57$\frac{17}{18}$: but this verſed ſine is = 13,324; therefore the velocity of the pendulum is = velocity acquired by falling perpendicularly 13,324 inches. And with this velocity the globe meets with a reſiſtance which is to its weight as ,52659 to 121, or (taking that part only of the reſiſtance which is as the ſquare of the velocity) as ,49505 to 121.

Alſo if a globe of water of equal magnitude moves with the ſame velocity, its reſiſtance will be to its weight as ,49505 to 213,4; or as 1 to 431$\frac{1}{15}$. Whence, in the time in which the globe uniformly deſcribes 26,648 inches, the weight of the globe of water will generate all that velocity in the falling globe; therefore the velocity deſtroyed by reſiſtance will be to that acquired by gravity (in the ſame time) as 1 to 431$\frac{1}{15}$, or velocity loſt = $\frac{1}{431\frac{1}{15}}$ of the whole velocity. And therefore, in the time it would uniformly deſcribe its ſemi-diameter with the ſame velocity, it would loſe the $\frac{1}{3341.7}$ part of its motion.

[P. 79, after five oſcillations] this certainly ſhould be after 10 oſcillations, as appears by the proceſs of the calculation.

[P. 80. So that the difference 0,4475, &c.] for the motion loſt is $\propto$ the reſiſtance, and that $\propto$ the ſquare of the velocity and ſquare of the time nearly, which in this caſe is a conſtant product.

[P. 81, l. 2.] Here is a ſmall error, viz. ,61675 inſtead of ,61705; whence 45,453 would have been 45,43, and the reſiſtances as 7,002 to 1.: but this is not material.

[P. 82, becauſe the reſiſtance, &c.] for the reſiſtance $\propto$ ſquare of the time × ſquare of the velocity, which product is invariable, becauſe equal arcs are always deſcribed.

[Pr. 32, ratio of denſity] he means any given ratio of denſity which are proportional to the particles.

[Pr. 32. Proportional times] any times, in that conſtant ratio; that is, let them move among one another in ſimilar directions, with velocities which are as the particles.

[Pr. 32, cor. 2, ſpaces proportional to their diameters] in theſe proportional times.

[Pr. 33, cor. 2, for if the forces, &c.] for the reſiſtance ariſes from the centrifugal forces, and from the colliſions of the particles. The reſiſtances of the firſt ſort are as the motive forces, that is, as the accelerative forces and quantity of matter; that is (by ſuppoſition), as the ſquares of the velocities and quantities of matter; that is (becauſe the quantities of matter are given, the fluid being the ſame), as the ſquares of the velocities.

The reſiſtances of the ſecond ſort are as the number of reflexions and their forces; that is (as is proved in the prop.), as the ſquares of the velocities, ſquares of the diameters, and denſities of the parts; that is (becauſe the diameters and denſities are given), as the ſquares of the velocities accurately.

[As alſo the bodies E and G;] let E and G be vaſtly ſwifter than D and F.

[Pr. 34, but the former of, &c.] for $bH \times CB = (BE^2 =) ObN$, which is the property of the parabola.

[Pr. 34, ſchol. leſs reſiſted than the former ſolid;] for if FG and IH be produced till they interſect, they will form a right angle; and therefore the fruſtum FGBHI will be leſs reſiſted than if the lines ſhould interſect in an obtuſe angle (as is ſhewn in the cone), which they would do if the lines fell within FG and IH; and for the ſame reaſon the reſiſtance would be greater in the curve itſelf, becauſe the lineola which conſtitute it will, when produced, interſect one another at obtuſe angles. What is mentioned beſides in this ſchol. is demonſtrated in the Appendix.

[Pr. 35, it follows, that the cylinder, &c.] for motion cylinder : motion medium : : cylinder's magnitude × denſity × velocity : (medium's magnitude × denſity × velocity =)

cylinder's $\frac{\text{magnitude}}{2}$ × 2 velocity × medium's denſity : : cylinder's denſity : medium's denſity.

[Ib. to the force by which] to the motive force by which its whole motion, &c.

[Pr. 35, cor. 2] for the reſiſtance of the globe and the force (that will quite take away its motion in the time it moves two or four thirds of its diameter) are in a given ratio; therefore increaſe the velocity of the globe in any ratio, and the force (that will utterly deſtroy all its motion in a given ſpace) will be increaſed in a ratio of the velocity directly and the time inverſely, that is, as the ſquare of the velocity; therefore the reſiſtance of the globe is increaſed in the duplicate ratio of the velocity.

[Ib. cor. 3] for the ſame reaſon, increaſe the diameter of the globe in any ratio, and the motive force (that will utterly deſtroy its motion) will be increaſed in the triplicate ratio of the diameter directly, and the time inverſely; that is, in the triplicate ratio of the diameter directly, and the ſimple ratio of the diameter inverſely; that is, as the ſquare of the diameter; and therefore the reſiſtance of the globe will be increaſed in the duplicate ratio of the diameter.

[Cor. 6, and its reſiſtance, &c.] Let BC be the reſiſtance at firſt, then reſiſtance at the firſt : reſiſtance at the end : : $BC^2 : EF^2$; but AE : AB : : BC : EF : : EF : BH; therefore $BC^2 : EF^2$: : BC : BH. ∴ BH is the reſiſtance at the end.

[Cor. 7. As the logarithm, &c.] For (by ſchol. prop. 86, Fig. 42, hyperbola) it appears that if AH = 1, AB = T, BK = t; then it will be CBKF : EHKF : : (logarithm T + t — logarithm T =) logarithm $\frac{T+t}{T}$: logarithm $\overline{T+t}$. And EHKF : AHED : : (logarithm $\overline{T+t}$: ,43429448 : :) logarithm $\overline{T+t}$ × 2,30258509 : 1; alſo AHED : CBKG : : HE : CB × t : : T : t. Therefore, *ex equo*, CBKF : CBKG : : logarithm $\frac{\overline{T+t}}{T}$ × 2,30258 × T : t : : logarithm $\frac{\overline{T+t}}{T}$ × 2,30258509 : $\frac{t}{T}$.

[Pr. 36, cor. 2.] Forces are equal when their effects are equal in a given time; but the effect of the motion of the effluent water (in the time a body deſcends through GI) = a cylinder whoſe length is 2GI, and baſe the hole EF; alſo the effect of the gravity or weight of a column whoſe height is GI, and baſe the hole EF, = (by reaſon of an accelerated motion, and in the ſame time) a cylinder whoſe length is GI, and baſe the hole EF; and therefore the effect of the preſſure of twice that column (in the ſame time) = a cylinder whoſe length is 2GI, and baſe the hole EF; = the firſt cylinder, *ergo*, &c.

[Cor. 3, or IH + IO to 2IH] for ih : ho : : io : og; or ho : og : : ih : io; and ho + og : 2ho : : ih + io : 2ih : : io + ig : 2io : : $\odot$ EF + $\odot$ AB : 2 $\odot$ EF.

[Ib. cor. 10.] This cor. ought to be more exactly computed and demonſtrated, becauſe the following propoſitions depend thereon.

[Pr. 37, p. 109. And is therefore nearly equal, &c.] This reſiſtance = weight of a cylinder (whoſe baſe is that little circle and altitude $\frac{1}{2}$IG, from which altitude the cylinder muſt fall, &c.) = force by which its motion may be generated, &c.

[Pr. 37, cor. 1.] Let F = force (mentioned in the prop.), M = medium's denſity, C = cylinder's denſity, V = velocity, D = diameter, T = time (of deſcribing four times its length), R = reſiſtance; then $R = F \frac{\times M}{C}$ (by the prop.)

$$\propto C \times \frac{V}{T} \times D^2 \times \frac{M}{C} \propto V^2 D^2 M.$$

[Cor. 2.] Let EF be given, and denſity of the medium be alſo given. Then becauſe this reſiſtance = weight of a cylinder (whoſe baſe is PQ, and height $\frac{1}{2}$IG) $\times \frac{EF^2}{EF^2 - \frac{1}{2}PQ^2}$, therefore weight of this cylinder = reſiſtance $\times \frac{EF^2 - \frac{1}{2}PQ^2}{EF^2}$. But the force requiſite to take away the motion of this cylinder, whilſt it moves four times its diameter $\propto$ weight of this cylinder $\times$ velocity $\times$ $\frac{1}{\text{time of deſcribing four times its length}} \propto$ reſiſtance $\times$ $\frac{EF^2 - \frac{1}{2}PQ^2}{EF^2} \times \frac{EF^2 - PQ^2}{EF^2} \times \frac{EF^2 - PQ^2}{EF^2}$. And the denſity of the cylinder being increaſed, the force is increaſed in the ſame ratio.

[Cor. 3] for ſince reſiſtance : force : : $EF^6 \times$ denſity medium : $\overline{EF^2 - \frac{1}{2}PQ^2} \times \overline{EF^2 - PQ^2}^2 \times$ denſity cylinder : : medium denſity : $\frac{\overline{EF^2 - \frac{1}{2}PQ^2} \times \overline{EF^2 - PQ^2}^2}{EF^6} \times$ cylinder denſity : increaſe the diſtance it moves in any ratio, and the force (to deſtroy the motion) will be decreaſed in that ratio, as ſuppoſe the ratio of $\frac{\overline{EF^2 - \frac{1}{2}PQ^2} \times \overline{EF^2 - PQ^2}^2}{EF^6}$. Then reſiſtance : force : : medium denſity : cylinder denſity.

[Schol. pr. 37, AE and BE (Fig. 43) be two parabolic arcs] ſince the reſiſtance is the ſame, whether the fluid or the cylinder move, therefore the caſe of the reſiſtance of the circle in Fig. 4 is the ſame caſe with the reſiſtance of the circle AB in Fig. 6, conſidering the velocities HG and EG, wherewith they are acted on; but (by cor. 9, pr. 36) the reſiſtance on PQ = ½ cylinder on PQ and height GH (through which a falling body acquires the velocity which the fluid has on PQ); but ½ this cylinder is = a parabolic ſpindle on PQ and height GH, nearly. Therefore the reſiſtance on AB = ½ cylinder on AB and height GE (= GH), through which a falling body acquires the velocity with which the cylinder moves, which is equal to a parabolic ſpindle on AB and height GE = AEB, by conſtruction [ſince *latus rectum* : (HG =) GE : : GE : AG.] This is the very conſtruction of Fig. 2.

[Ib. muſt be to this force as 2 to 3, at leaſt,] for the weight upon the little circle (in cor. 7, p. 28), is there ſhewn to be equal to a cylinder of ⅓ the height GH at leaſt. But the weight of a cylinder on that circle and ½ that height is the force whereby the cylinder's motion may be generated in the time it moves four times its length, or 2GH; therefore the reſiſtance in that cor. is to the weight here demonſtrated *(cæteris paribus)*, that is, to the force : : as $\frac{1}{3}$ to $\frac{1}{2}$: : that is, as 2 to 3, at leaſt; and increaſing the denſity, the force is increaſed in the ſame ratio.

[Ib. let CF and DF.] It is neceſſary to allow as much time for the meeting of the particles of water (at the axis of the ſolid from the ſide thereof), after they are paſt the ſolid, as for their ſeparation before they come to it. Now ſuppoſe this to be the caſe of Fig. 4. In the time that a particle paſſes from H to P, the ſame particle with the velocity in P will in the ſame time deſcribe 2HG, and therefore muſt paſs twice as far to arrive to the axis again. Whence, in Fig. 6, the particle F muſt be twice as far from CD as E from AB; but the height of FCD = 2 height of AEB, by conſtruction. For let z, y = 2 ordinates to x = $\frac{1}{2}$ AB; and zz = 4rx = 4yy, and z = 2y.

[Cor. 1, pr. 38.] the demonſtration is the ſame with that of cor. 1, pr. 37.

[Cor. 2, pr. 38.] Let denſity of globe : denſity of fluid : : d : 1; then d × $\frac{4}{3}$ diameter of the globe = ſpace fallen, and d × $\frac{8}{3}$ diameter globe = ſpace it deſcribes afterwards, in the time of its fall. Then, becauſe forces (which generate a given motion) are reciprocally as the (times, that is, reciprocally as the) ſpaces which any body deſcribes with the ſame uniform velocity, therefore the force of the globe's comparative weight (which generates its motion in the time it deſcribes d × $\frac{8}{3}$ diameter) : force that generates the ſame motion (while it deſcribes $\frac{8}{3}$ diameter) : : $\frac{8}{3}$ diameter : d × $\frac{8}{3}$ diameter : : 1 : d : : denſity of the fluid : denſity of the globe. Therefore, &c.

[Cor. 3, pr. 38.] The velocity being given, the motion M is alſo given; but (by this prop.) reſiſtance × d = force which will generate the motion M (in the time of moving $\frac{8}{3}$ diameter); but this force muſt be = reſiſtance, which will quite take away the ſaid motion M (in the ſame time of moving $\frac{8}{3}$ diameter). Now augment the time of moving (preſerving the velocity) in any ratio (let it = T); and this laſt reſiſtance (which will quite take away the ſaid motion M in the time T) will be decreaſed in the ſame ratio (= R). Therefore T and R are alſo given. Therefore by cor. 7, &c.

[Cor. 4, prop. 38.] for let M = $\frac{8}{3}$ diameter in the time T; and becauſe (by ſup.) $\frac{tM}{T+t} = \frac{1}{2}M$, therefore T = t. Now (by cor. 7, pr. 35) it will be $\left(\frac{t}{T} =\right)$ 1 : (log. $\frac{T+t}{T} \times 2$, 3, &c. = log. 2, × 2, 3, &c. =) ,693 : : $\frac{8}{3}$ diameter : $\frac{5.545}{3}$ diameters, which is leſs than two diameters. Now that M ought to be = $\frac{8}{3}$ diameter in the time T is plain; for it is required (by cor. 7, pr. 35) that the reſiſtance R quite takes away M in the time T. But (by this pr. 38) the reſiſtance (= force required) will juſt take away the motion M in moving $\frac{8}{3}$ diameter. But if T be any other time than that of moving $\frac{8}{3}$ diameter, then the force to deſtroy M in that time will not be = reſiſtance, but greater or leſſer than it.

[Pr. 40, let the globe, &c.] This is calculated in prob. 14 of my book of Fluxions. In cor. 2 of that prob. G, H, and N, are found, and thence the velocity = $\frac{N-1}{N+1}$ H, where H is the greateſt velocity the globe can acquire in falling in the fluid; and the height fallen comes out the ſame in value with his, but in other ſymbols; but the one may be eaſily reduced to the other.

[Ib. the numbers in the fourth column, &c.] 2F is the ſpace deſcribed in the time G with the greateſt velocity H (by the prob.); therefore G : 2F : : P: $\frac{2P}{G}$ F = the ſpace deſcribed in the time P with the greateſt velocity, the numbers in the fourth column.

[Sch. pr. 40, and any other globe, &c.] that is, the magnitude of any body $\propto$ the exceſs of its weight in vacuo above that in water. Or 132,8 : 1 : : this ſaid exceſs : magnitude of the globe. Hence diameter globe (in Exp. 1.) = 0,84224.

[Ib. exp. 1, but this ſpace, by reaſon, &c.] to demonſtrate this, it is neceſſary to premiſe, that the greateſt velocity wherewith a globe can deſcend by its comparative weight through a fluid in a canal is that which it may acquire by falling with the ſame weight, without reſiſtance, and in its fall deſcribing a ſpace which is to $\frac{4}{3}$ diameter as mn^2d to l^3. Putting denſity globe : denſity fluid : : $d : 1$, and $l =$ orifice of the veſſel, $N = l —$ great circle of the globe, $m = l — \frac{1}{2}$ great circle. For (as in cor. 2, pr. 38) the ſpace it afterwards deſcribes (in the ſame time of its fall) $= \frac{mn^2d}{l^3} \times \frac{8}{3}$ diameter; then the force of the globe's comparative weight (which generates its motion in the time it deſcribes $\frac{mn^2d}{l^3} \times \frac{8}{3}$ diam.) : force (that generates the ſame motion in the time it deſcribes $\frac{8}{3}$ diameter) : : ($\frac{8}{3}$ diameter : $\frac{mn^2d}{l^3} \times \frac{8}{3}$ diameter : :) $l^3 : mn'd$; therefore (by pr. 39) the reſiſtance of the globe = force of the globe's comparative weight (which generates this motion in the time it deſcribes $\frac{mn^2d}{l^3} \times \frac{8}{3}$ diameter, or falls $\frac{mn^2d}{l^3} \times \frac{4}{3}$ diameter); and, therefore, this force cannot accelerate the globe. Q.E.D. Now,

By the experiment, $2F = d \times \frac{8}{3}$ diameter; but the ſpace the globe uniformly deſcribes (with the greateſt velocity wherewith it can deſcend in the canal or veſſel, and in the time of acquiring it with its compound weight) $= \frac{mn^2d}{l^3} \times \frac{8}{3}$ diameter $= \frac{mn^2}{l^3} \times 2F = \frac{mn^2}{l^3} \times 4.4251$. Alſo $\sqrt{95{,}219} : \sqrt{\frac{mn^2}{l^3} \times F} :: 1'' : \sqrt{\frac{mn^2}{l^3}} \times \sqrt{\frac{F}{95{,}219}} = \sqrt{\frac{mn^2}{l^3}} \times G = \sqrt{\frac{mn^2}{l^3}} \times 0{,}15244$ (by the laws of falling bodies) = time of acquiring the greateſt velocity. And therefore $\left(\sqrt{\frac{mn^2}{l^3}} \times G\right.$

(= time) : $\frac{mn^2}{l^3} \times 2F$ (= ſpace) : :) G : $\sqrt{\frac{mn^2}{l^3}} \times 2F$: : 4″ : $\frac{n}{l}\sqrt{\frac{m}{l}} \times \frac{8F}{G}$ = (ſpace deſcribed in 4″) = $\frac{n}{l}\sqrt{\frac{m}{l}} \times$ 116,1245. Subduct $\frac{mn^2}{l^3} \times 1.3862944F = \frac{mn^2}{l^3} \times 3{,}0676$; and $\frac{n}{l}\sqrt{\frac{m}{l}} \times 116{,}1245 - \frac{n}{l}\sqrt{\frac{m}{l}} \times 3{,}0676$ = ſpace fallen in the fluid, $= \overline{116{,}1245 - 3{,}0676} \times \frac{n}{l}\sqrt{\frac{m}{l}}$, nearly, as the author makes it.

[Ib. ex. 13.] This and the following are upon the ſame computation as the foregoing experiments in water; to wit, the theory of non-elaſtic fluids: the reaſon is, becauſe he conſiders fluids as elaſtic whoſe parts are diſcontinued, &c. See ſchol. pr. 35.

[Ib. p. 128, equal to $\frac{1}{4586}$, &c.] This is plain from what follows afterwards; for let a ſpace : $\frac{8}{3}$D : : 860 : 1; this ſpace $= \frac{6880}{3}$ D = 2293D, and this ſpace the globe moves in the time T with velocity V; let, therefore, T = 2293, and t = $\frac{1}{2}$. Then motion loſt $= \frac{\frac{1}{2}V}{2293 \times \frac{1}{2}} = \frac{1}{4587}$ V. Note; t (= $\frac{1}{2}$) is not ſenſibly increaſed by the reſiſtance in moving through $\frac{1}{2}$D; and hence I have taken it for the ſame.

[Ib. let D be the, &c.] for, put denſity globe : denſity fluid : : d : 1; alſo let the ſpace $\frac{8d}{3}$ D = S. Now, by prop. 38, d × reſiſtance = force, which (uniformly continued) will quite deſtroy its motion in the time it moves $\frac{8}{3}$D; therefore d × reſiſtance will quite take away its motion whilſt it moves $\frac{8}{3}$D. But (becauſe the force of that reſiſtance and the time of its acting are reciprocally proportional) the reſiſtance the globe meets with will quite take away its motion whilſt it moves (with V) $\frac{8d}{3}$ D = S. Therefore (by cor. 7, pr. 35) the globe in the time T (of moving S) meets with a reſiſtance R which will quite deſtroy its motion V; therefore in the time t it will loſe $\frac{t}{T + t}$ V, &c.

SECTION VIII.

[Pr. 42, and (Fig. 44) becauſe the motion of the waves, &c.] Let the waves A, B, C, move from A towards C, and ſuppoſe the water to deſcend from the vertex C into the *hollow* E, and thence up to B, which it will do, ſuppoſing the vertices A, B, C, to be of equal altitude; alſo let the water deſcend from B to D, and aſcend to A, &c. Now ſince the water in the vertex B deſcends towards D at the ſame time that other water (that had deſcended from C to E) is aſcending to B, therefore the water will be lower at the point B and higher at the point b than before; and conſequently that the vertex of the wave B will be transfered towards C. Therefore the vertices (and hollows, and any correſponding points) of all the waves will move from thence towards C; therefore the waves will be carried on by the deflux of water down CE, BD; and the accumulation thereof on EB, DA.

[Pr. 43, caſe 1, at nearly equal diſtances, &c.] they ſet forward at equal diſtances, and continue ſo, becauſe they move with equal velocity, as appears by (the demonſt. of caſe 1, 2, 3, of) prop. 48. (Fig. 45.)

[Pr. 46, becauſe the motion of the waves is carried on, &c.] Let A be the vertex of a wave. The fluid at b is more preſſed (by reaſon of a greater depth of incumbent water) than d; therefore that greater preſſure cauſes the water to recede from b in all directions, and to aſcend at c; therefore the vertex of the wave is now in c. Alſo a greater preſſure of water propagated from d to f cauſes the water to deſcend at c, and aſcend at e; therefore the vertex of the wave moves along a, c, e, &c. And this it will always continue to do, if at firſt any force is ſuppoſed to be applied to the ſide gha, to hinder the aſcent of the water there in like manner as it aſcends at c, e; for then the ſide gha will be afterwards always deſcending, and the ſide ace aſcending, without the continuance of ſuch force.

[Ib. tranſverſe meaſure] is meaſuring along the ſurface of the water; as is plain from the demonſtration.

[Ib. cor. 2, rather performed in a circle] as in the note on prop. 42 (Fig. 46). But if the breadth of the waves be meaſured not in the right line acb, but along the ſurface of the water (becauſe the deflux of water through b, c, a, is made in a curve line near a circle), then the time mentioned in the prop. will be very nearly as it is there aſſigned.

[Pr. 47] Fig. 1, Pl. 9, is wrong. It ſhould be thus (Fig. 47); [and the original is wrong, too] where l, m, n, ſtand at N, M, L.

[Ib. — with thoſe of an oſcillating pendulum.
Ib. according to the law of a pendulum oſcill.] for, by prop. 52, lib. 1, ſpace deſcribed by a pendulum in the time PI is to a whole vibration as the time PI to the time PIS; and to two vibrations as PI to PHSP; and the accelerating force of a pendulum is as its diſtance from the loweſt point. See cor. pr. 51.

[Ib. that is, (becauſe HL — KN is,) &c.] that is, HL — KN : HK : : OM : OI.

[Pr. 48, caſe 1, diſtances of the pulſes, &c.] the diſtances of the pulſes in one medium be = diſtances of pulſes in the other.

[Ib. and therefore the pulſes;] hence it appears that the correſpondent parts in both mediums (vibrate or) go and return together, or in equal times, though one makes longer vibrations than the other. But (by ſuppoſition) the diſtances of pulſes in both mediums are equal; and the pulſes in both are tranſlated through theſe equal diſtances in the times of the parts of the mediums (vibrating or) going and returning, which times are proved equal; therefore the pulſes in both mediums move equal diſtances in equal times, and therefore are equally ſwift.

[Pr. 48, caſe 2, then will their contractions, &c.] be equal, in equal ſpaces.

[Ib. and moreover, &c.] for ſpace $\propto \frac{\text{ſquare of the time}}{\text{matter}}$ for theſe ſpaces are generated by accelerative forces), and

$\text{time} \propto \sqrt{\text{ſpace}} \times \sqrt{\text{matter}} \propto \sqrt{\text{ſpace} \times \text{ſpace}} \propto$ ſpace.

[Ib. caſe 3, the time which is neceſſary, &c.] for ſpace $\propto \frac{\square\,\text{time} \times \text{force}}{\text{matter}} \propto \frac{\square\,\text{time} \times \text{force}}{\text{denſity}}$. Therefore if the ſpace be given, time $\propto \sqrt{\frac{\text{denſity}}{\text{force}}}$. Therefore velocity $\propto \frac{1}{\text{time}} \propto \sqrt{\frac{\text{force}}{\text{denſity}}}$.

[Pr. 49. — ratio of PO to A conjunctly;] that is, time $\sqrt{\frac{\text{matter}}{\text{force}}} \propto \frac{1}{\sqrt{\text{force}}}$, when the quantity of matter is given (this in accelerated motions). Then time of one vibration by that elaſtic force : time of one vibration of a pendulum PO, by the weight : : $\sqrt{VV}$: $\sqrt{PO \times A}$; and time of one vibration of PO, by the weight : time of one vibration of A : : $\sqrt{PO}$: $\sqrt{A}$. Therefore, &c.

[Ib. — of the going and returning] of the pendulum PO.

[Ib. — of one oſcillation compoſed of the going and returning,] of the pendulum A.

[Ib. cor. 1, — to its circumference.] For (Mechanics, prop. 24) diameter : circumference : : time of fall through ½ radius : time of one vibration. Therefore as radius : circumference : : time of fall through ½ radius : time of two vibrations.

[Ib. cor. 2.] for velocity $\propto \frac{\text{ſpace}}{\text{time}} \propto \frac{A}{\sqrt{A}} \propto \sqrt{A} \propto \sqrt{\frac{\text{e. force}}{\text{denſity}}}$. By elaſtic force he means that which ariſes by heat, as well as that ariſing from the compreſſion.

[Pr. 50. Let the number] of the double vibrations, &c.

[Ib. ſchol. in the ſubduplicate ratio of the defect of the matter] that is, of the quantity of vapour or watery particles And (by prop. 49, cor. 2) the velocity will be increaſed as (the $\sqrt{\text{denſity}}$, that is, as) the $\sqrt{\text{quantity of true air in a given ſpace}}$ is decreaſed.

[Ib. near 100 pulſes in] This computation pre-ſuppoſes that thoſe vibrations of the ſtring are double vibrations, or vibrations compoſed of its going and return.

SECTION IX.

[Pr. 51, but the differences of the angular motions, &c.] theſe are as the difference of the abſolute motions round the axis; that is, as the difference of the abſolute tranſlations, or as the relative tranſlations (which are thoſe here ſpoken of) directly, and the diſtances inverſely.

[Ib. of quadratures of curves, &c.] Let SD $= x$; Dd $= y$; then Dd $\propto \frac{1}{xx} \left(= \frac{b}{xx} \right)$, by conſtruction; therefore $y = \frac{b}{xx}$; and $y\dot{x} = bx^{-2}\dot{x}$; and $\frac{bx^{-1}}{-1} = \frac{-b}{x} =$ area; therefore area $\propto \frac{1}{x} =$ DdQ.

(Ib. cor. 3.] In a given time let the parts 1, 2, 3, of the fluid deſcribe the (Fig. 48) ſpaces 1r, 2s, 3t; take away or add the equal angular motions 1n, 2o, 3p; ſo that the ſpaces nr, os, pt, be deſcribed in that given time: now whether the points 1, 2, deſcribe the ſpaces 1r, 2s, or nr, os, the tranſlations at the end of the given time will be = si; alſo if 2, 3, deſcribe 2s, 3t, or os, pt, the tranſlations at the end of the given time will be the ſame in either caſe, viz. = ut; therefore the tranſlations being equal, and the attrition alſo, the motions will be continued.

[Pr. 52, cor. 8, about any given axis] paſſing through the globe; the relative motion of the parts of the globe and fluid are the ſame ſtill: therefore, &c.

[Ib. cor. 9, ſemi-diameter of the globe;] let BC be a plane (Fig. 49), D a point of the veſſel, A a point of the globe; then period. time of A : time B : : $<$ velocity B : $<$ velocity A; and therefore time A + time B : time A : : (velocity B + velocity A =) $<$ velocity A from B : $<$ velocity B : : time B : time A from the plane BC (to BC again) : : $CB^2 : CA^2$. But time B from DC = time D from BC; therefore time D (from BC) : time A (from BC) : : $CB^2 : CA^2$; as it ſhould be by the prop. and cor. 8.

BOOK III.

[Pr. 3, the action of the ſun, attracting, &c.] ſee cor. 7, pr. 66, lib. 1, or cor. 14, where KL is $\propto$ PT, nearly (SK or ST being given); and the force TM in its mean quantity is = PT, nearly.

[Pr. 4, and the ſpace which a heavy body deſcribes] by falling in 1″ : half the length of the pendulum (= ½ radius) : : ſquare of 1″ : ſquare of the time of falling through half the pendulum : : (Mechanics, pr. 24) ſquare of the circumference : ſquare of the diameter.

[Ib. p. 169, line 17, the mean diſtance of 60 diameters] it ſhould be 60 ſemi-diameters, as it is in the original.

[Pr. 6, p. 174, — ſubduplicate of that proportion, as by ſome computations I have found,] i. e. as I found by making ſome calculation.

For let R be the diſtance of the Sun and Jupiter, d the greater force, e the leſſer; then, to find the diſtance x where d may be diminiſhed to e: Becauſe the forces are reciprocally as the ſquares of the diſtances, ſay $\frac{1}{R^2} : \frac{1}{x^2} : : d : e : : x^2 : R^2$; and $x : R : : \sqrt{d} : \sqrt{e}$; therefore at the diſtance x the weight d (of the ſatellite) would become equal to e; and the centre of their orbits would not be in Jupiter, contrary to experience. And if the force of the ſatellite was leſs, by a like computation, the centre of their orbits would be nearer than Jupiter, alſo contrary to experience.

[Pr. 8, cor. 1, and $\frac{1}{169282}$ reſpectively] this ſhould be $\frac{1}{196282}$, or rather $\frac{1}{194075}$.

[Ib. at the diſtances 10000, &c.] theſe are as the diameters; and the apparent diameters are found by aſtronomical obſervation, which, × by the proportioned diſtances, gives the ratio of the real diameters.

[Pr. 8, cor. 4, — of ſo much the greater denſity.] This is not generally true, as appears by cor. 2 and 3.

[Ib. cor. 3, — truly defined.] This is demonſtrated in Graveſand, l. IV. p. 232-3. See Appendix, p. 20.

[Pr. 10, loſe almoſt a tenth part of its motion.] For, by ſchol. pr. 40, p. 128, it is, 1 : 860 : : $\frac{8}{3}$: 2293.33 † = ſpace deſcribed in the time T; putting D = 1, V = 1; and T : t : : 2293.33 : 2293.33 $\frac{t}{T}$ = ſpace (uniformly) deſcribed in the time t. Then (p. 129) $\frac{t}{T}$: 2,302, &c. × log. $\frac{T + t}{T}$: : 2293.33 $\frac{t}{T}$: 229.5; therefore 2293.33 × 2.30258 × log. $\frac{T + t}{T}$ = 229.5; and log. $\frac{T + t}{T}$ = ,043461; and therefore $\frac{T + t}{T}$ = 1.105†; and $\frac{T}{T + t} = \frac{1}{1.105} = \frac{9}{10}$ †, and $\frac{t}{T + t}$ = $\frac{1}{10}$, nearly.

[Ib. or as 75 (0^{12}) to 1 nearly.] In Fig. 3, Pl. 5, l. II. Let SA = rad. earth = 20949674 feet, AB = 1, AF = 200 miles = 1056000 feet; then, by cor. pr. 22, II. Aa — Bb : Aa — Ff : : thin : thnz; that is, AB × SF : AF × SB : :, or 1 : 1005325 : : thin : thnz : : (by ſch. pr. 86, hyperbola) log. AH — log. BI : log. AH — log. FN; becauſe water is 860 times denſer than air, and the incumbent weight of the atmoſphere at the ſurface of the earth is about = weight of 33 feet of water; therefore the weights (and conſequently the denſities) in A and B are as 28380 and 28379; whence AH or St = 28380, and BI or Sn = 28379; and log. AH — log. BI = ,0000152, log. AH = 4,4530124; therefore 1 : 1005325 : : ,0000152 : 4,4530124 — log. FN; and log. FN = — 11,1730. And therefore FN = ,000000000015, and denſity in F : denſity in A : : 15 : 28380000000000000 : :, or as 1 : to 1800000000000000, which is leſs than 1 to 75000000000000; but the computation will vary as the radius of the earth, the denſity of water and air at the earth, is ſuppoſed to vary.

[Ib. and hence the, &c.] as before (by pr. 40, ſchol.), ſince denſity of water : to denſity of air 200 miles from the earth : : 645 (0^{14}) : 1. Space deſcribed in time T = 172 (0^{15}). And

$\frac{t}{T}$: log. $\frac{T+t}{T} \times 2.302$, &c. : : 172 $(0''5) \times \frac{t}{T}$ (= ſpace deſcribed in time t) : 2792250000 = ſpace Jupiter deſcribes in 1000000 years. Therefore log. $\frac{T+t}{T} = ,00000000705$. And $\frac{T+t}{T} = 1.000000017$; and $\frac{T}{T+t} = ,999999986$; and $\frac{t}{T+t} = ,000000014$, which is leſs than $\frac{1}{1000000}$.

[Pr. 13, are almoſt as 16, &c.] for (by this and cor. 2, pr. 8) accelerative force of Jupiter towards Saturn : accelerative force of Jupiter towards the Sun : : $25 \times \frac{1}{3021} : 16 \times 1 : : 81 ; \frac{16 \times 81 \times 3021}{25}$.

[Pr. 14, ſchol. And hence we may find, &c.] For, by cor. 7, pr. 66, I. the apſides of their orbits move *in conſequentia.* And, by cor. 16, of the ſame, the motion of the apſides of the body P (Pl. 21, Fig. 2) will be as the periodical time of P directly, and the ſquare of the periodical time of T inverſely; that is, as the periodical time of P (becauſe the periodic time of T round S, or of the Sun round Jupiter or Saturn, or (which is the ſame thing) of Jupiter or Saturn round the Sun, is given); that is (by prop. 15, I.), in the ſeſquiplicate ratio of their diſtances PT.

[Pr. 19, in the duplicate proportion of the rad. to the coſine of lat.] (Fig. 50) for (by cor. 3, pr. 4, l. I.) centrifugal force at the equator : centrifugal force at a from the axis (= ab) : : rad. : coſ. lat. : : r : c. Alſo the force *directly* from the axis : force directly from the earth : : ab : be : : r : c. Therefore, *ex equo,* centrifugal force at the equator : centrifugal force directly from the earth, at a : : rr : cc.

[Ib. but by computation (from] Let (Fig. 51) GC = t = 101, BP = C = 100, PE = x. Then will $ED^2 = \frac{tt}{cc} \times \overline{cx - xx}$. And $ER^2 = PD^2 = \frac{ttx}{c} - \frac{tt}{cc}x^2 + x^2$. And $ER = \sqrt{\frac{tt}{c}x + \frac{cc - tt}{cc}x^2}$ = (ſuppoſe to) $\sqrt{r^2x - sx^2}$.

Whence the fluxion of the area PQRE is $= \overline{r^2x - sx^2}|^{\frac{1}{2}}\dot{x} = rx^{\frac{1}{2}}\dot{x} - \frac{sx^{3}}{2r}\dot{x} - \frac{s^2x^{\frac{5}{2}}}{2.4r^3}\dot{x} - \frac{3s^3x^{\frac{7}{2}}}{2.4.6r^5}\dot{x} - \frac{3.5s^4x^{\frac{9}{2}}}{2.4.6.8r^7}\dot{x} -$ &c. And the area PQRE $= \frac{2}{3}rx^{\frac{3}{2}} - \frac{sx^{\frac{5}{2}}}{5r} - \frac{s^2x^{\frac{7}{2}}}{7.4r^3} - \frac{3s^3x^{\frac{9}{2}}}{9.4.6r^5} - \frac{3.5.s^4x^{\frac{11}{2}}}{11.4.6.8r^7} -$ &c. $= \frac{2}{3}rx^{\frac{3}{2}} - \frac{3.sx}{2.5r^2}A + \frac{5.sx}{7.4r^2}B + \frac{7.3.sx}{9.6r^2}C + \frac{9.5.sx}{11.8.r^2}D +$ &c. $=$ (putting $\frac{sx}{rr} = q$) $\frac{2}{3}rx^{\frac{3}{2}} - \frac{3}{10}Aq + \frac{5}{28}Bq + \frac{7}{18}Cq + \frac{45}{88}Dq +$ &c. $=$ (when $x = c = 100$; and s being $= \frac{cc - tt}{cc} = ,0201$; and $r^2 = \frac{tt}{c} = 102,01.$ $=$) to 6693,39. And PBM = 5000. Whence PQRM = 1693,39. Therefore, by cor. 2, pr. 91, I. force of the ſpheroid : force of the ſphere : : (when P and A coincide) $AS - \frac{AS \times PQRM}{CS^2} : \frac{AS}{3} : : 1 - \frac{PQRM}{CS^2} : \frac{1}{3} : : 1 - \frac{4 \times 1693.39}{10201} : \frac{1}{3} : : 1,008 : 1 : : 126 : 125.$

[Ib. p. 190, if the denſity, &c.] this will appear by conſidering what went before, and cor. 3, pr. 91, I.

[Ib. but if the diurnal, &c.] for the (centripetal or) centrifugal force is as the ſquare of the velocity when the rad. is given.

[Ib. 191, — augmented in proportion as the force of gravity is diminiſhed] for, reviewing the former calculation, if the denſity of the earth were greater than it is, the force of gravity to the centrifugal force would be greater than in that proportion of 289 to 1 (and the two diameters would be in a leſs proportion than 289 to 288; and alſo the diameters would be in a leſs proportion than 230 to 229); conſequently the ratio of the difference of the diameters to either decreaſes as the gravity increaſes; and, on the contrary, that ratio increaſes as the gravity decreaſes. And upon this depends the foregoing proportion, as $\frac{4}{505} : \frac{1}{100} : : \frac{1}{289} : \frac{1}{229}$.

[Ib. line 15] read $\frac{29}{5} \times \frac{400}{94\frac{1}{2}} \times \frac{1}{229}$ to 1. In the original, 2d edit. it is $\frac{29}{5} \times \frac{5}{1} \times \frac{1}{229}$ to 1, or as 1 to 8.

[Pr. 20 (Fig. 52). Whence ariſes this theorem] for let nkyn be a circle circumſcribed round the ellipſis nhyl; then khm : fbc (=) abd : : $kt^2 : fg^2$. And ab $= \frac{khm \times fg^2}{kt^2 \times bd}$. But, becauſe khm and kt are given, and bd = (nearly to) 2kt, which is given, therefore ab $\propto fg^2$: but the difference between the weight at n and a is as (at — bt =) ab. Therefore the increaſe of weight is as fg^2, or the ſquare of the ſine of lat. at a, nearly, or (by trigonometry) as the verſ. of 2an.

[Ib. and the arcs of the degrees, &c.] The length of a degree is $\propto$ rad. of curvature, and in the points n and h are (by ex. 1, pr. 19, ſect. II. curve lines) as the parameters of ny and hl; that is, as hl^3 to ny^3, or 229^3 to 230^3; that is, as 56637 to 57382, as is inſerted in the table, p. 247.

But to find the rad. curvature at b, we have (by ex. 2, pr. 5, ſect. II. Fluxions) $mH = y + \frac{4 \times \overline{a - b}}{abb} y^3$; and, therefore (putting s for the ſine of the angle ntb), rad. curvature mC, at b $= \frac{y}{s} + \frac{4 \times \overline{a - b}}{abbs} y^3 =$ (becauſe $\frac{y}{s}$ is a given ratio) $R + \frac{4 \times \overline{a - b}}{abb} Ryy$, very near, R being the radius of curvature in n; and, therefore, the increaſe of this radius, which is as the increaſe of the degrees, is as yy or ss nearly. And here a — b being very ſmall, $\frac{4 \times \overline{a - b}}{abb} R$ is an extremely ſmall quantity.

[Pr. 23, — which I cannot here deſcend to explain.] Since the Moon's orbit is more eccentric than thoſe of Jupiter's ſatellites, the motion of the Moon's apogee will be greater in proportion than the motion of the apogee in any ſatellite.

[Pr. 27. The area, &c.] (Fig. 53) For the moment of the area is $\propto$ cb × ad. But ad $\propto$ angle bcd × cb = horary motion $\times$ cb. Therefore area $\propto cb^2 \times$ horary motion.

[Pr. 28. But the attraction of the moon;] for let A = attraction of P towards T. Let the attraction or force LS be reſolved into the two, LM, SM, the firſt acting towards M, the latter towards S. Then the whole attraction of P towards T is $A - ST + LM - TM$. Let $A - ST = F$. Then the attraction of P towards T in the ſyzygies is $F - 2AT$; and in the quadratures $F + CT$. By prop. 25, F : ML : : 178725 : 1000. And the forces 2AT and CT will be to each other as 2000AT and 1000CT; that is, as $\frac{2000}{CT \times N}$, and $\frac{1000}{AT \times N}$. And the force F in the ſyzygies and quadratures will be as 178725 directly, and the ſquare of the diſtance from T reciprocally. Therefore, &c. (Note, $+ \frac{2000}{CT \times N}$ is falſely printed for $- \frac{2000}{CT \times N}$, in line 11, p. 210.)

[Ib. — Quadrature in C; or, which —] for CTP : CTp : : angular motion of the moon from the ſun's quadrature : to its angular motion from the fixed point C; that is (becauſe the given circumference, or one revolution, is to be deſcribed in either caſe), : : periodic revolution : ſynodic revolution.

[Ib. But by computation I find,] with (Fig. 54) the radii TA, TC, deſcribe the circles AIc, Cic; and let AZ, Cz, touch the ellipſis in A and C; and take AI to Ac, and Ci to Cc, as $<$ CTP to CTp (Fig. 5, Pl. 10). Now in order to determine the curvature of the orbit, ſuppoſe the point a to coincide with A, and draw the parts of the orbit ARO, Cro, which will paſs between the ellipſis and circle; for TO or TE is leſs than TL, and greater than (TA or) Tc; and To (that is, Te) is leſs than TC or Tc, and greater than TI. Draw the lines TB, Tb, ſo that AB = Cb; and let the points A, I, C, and alſo C, i, c, coincide; and the curvatures of the ellipſis, orbit, and circles, at the points A, C, will be reſpectively as BE, BR, BI, and be, br, bi. And the difference of curvatures of the ellipſis and circles, and of the orbit and circles in A, C, will be as EI, RI, and ei, ri, reſpectively. But RI : EI or OC (becauſe TE = TO, and TI = Tc : :) $AI^2 : Ac^2$ (lem. 11, I.) : : $CTP^2 : CTp^2$. Whence $RI = EI \times \frac{CTP^2}{CTp^2} = \overline{BI - BE}$

$\times \frac{CTP^2}{CTp^2}$. Alſo ri : oc, or ei, or be — bi : : $Ci^2 : Cc^2 :: CTP^2 : CTp^2$. Whence ri $= \overline{be - bi} \times \frac{CTP^2}{CTp^2}$. But in the ⊙ TA, BI, AI, or AB, and 2AT, are ∺; and in the ⊙ TC, bi, Ci or Cb, 2CT, are ∺; whence BI $= \frac{AB^2}{2AT}$, and bi $= \frac{Cb^2}{2CT}$. Alſo in the ellipſis BE $\times$ 2AT : $BA^2 :: AT^2 : TC^2$; and be $\times$ 2CT : $bC^2 :: CT^2 : AT^2$. Whence BE $= \frac{BA^2 \times AT}{2CT^2}$; and be $= \frac{bC^2 \times CT}{2AT^2}$. Now from hence the curvature of the orbit at a : curvature of the orbit at C : : BR, or BI — IR : br or bi + ir $:: \frac{AB^2}{2AT} - \overline{BI - BE} \times \frac{CTP^2}{CTp^2} : \frac{Cb^2}{2CT^2} + \overline{be - bi} \times \frac{CTP^2}{CTp^2} :: \frac{AB^2}{2AT} \times \frac{CTp^2}{CTP^2} - BI + BE : \frac{Cb^2}{2CT^2} \times \frac{CTp^2}{CTP^2} + be - bi :: \frac{AB^2}{2AT} \times \frac{CTp^2}{CTP^2} - \frac{AB^2}{2AT} + \frac{BA^2 \times AT}{2TC^2} : \frac{Cb^2}{2CT^2} \times \frac{CTp^2}{CTP^2} + \frac{bC^2 \times CT}{2AT^2} - \frac{Cb^2}{2CT} :: \frac{CTp^2}{AT \times CTP^2} - \frac{1}{AT} + \frac{AT}{TC^2} : \frac{CTp^2}{CTP^2 \times CT^2} + \frac{CT}{AT^2} - \frac{1}{CT} :: \frac{CTp^2 - CTP^2}{AT \times CTP^2} + \frac{AT}{TC^2} : \frac{CTp^2 - CTP^2}{CT \times CTP^2} + \frac{CT}{AT^2} :: AT^3 + \frac{CTp^2 - CTP^2}{CTP^2} \times AT \times CT^2 : CT^3 + \frac{CTp^2 - CTP^2}{CTP^2} \times CT \times AT^2$.

[Pr. 29. The tangent of the angle (Fig. 55) CTP, &c.] On the axis DC deſcribe the circle CdD, and let the line Tb revolve uniformly to d in the ſame time that TP revolves to A, and deſcribes areas proportional to the times in the ellipſis CAD. Draw be ∥ to dT, and draw Tb, TP. Since the circle and ellipſis are deſcribed in equal times, parts proportional to the whole will be deſcribed in any equal times. Now area ellipſis : area circle : : TA : Td (= TC) : : eP : eb : : (ePC + ePT =) TPC : (ebC + ebT =) TbC; therefore the point in the circle is at b when the point in the ellipſis is at

P. But TA : (Td =) TC : : eP : eb : : tangent $\angle$ eTP : tangent $\angle$ eTb (= $\angle$ of the mean motion).

[Ib. Square of the ſine of the angle CTP.] Theſe things are plain from prop. 26, where Pd (Fig. 4) is the exceſs of the moment, and is as $\frac{PK^2}{PT}$.

[Ib. which we may effect, &c.] (Fig. 56) Let A be the area deſcribed by the moon in any time, arc CP = z, r = radius TC, PK = y, TK = u, t = tan. PTC. Then $\dot{A} = \frac{yy\dot{z}}{r}$ $= y\dot{x}$, and A = Fl. : $y\dot{x}$ = area CPK. But TPC expreſſes the mean motion, and therefore TPK is the equation, which is as TK × PK, or uy.

But, by the nature of the circle, $\dot{z} = \frac{rr\dot{t}}{rr + tt}$; and ſince $\dot{t}$ is in a given ratio to t (or as .3123 to 69), therefore $\dot{z}$ is as $\frac{rrt}{rr + tt}$ or $\frac{t}{\sqrt{rr + tt}}$ × TK, or y × TK, or as uy. Therefore, decreaſing the tangent in the ſubduplicate ratio of 11073 to 10973, or in the ſimple ratio of 69 to 68, 6877; accelerates the area in proportion of PK^2, as it ought to do.

[Ib. — in a proportion compounded of the duplicate, &c.] for (by cor. 16, pr. 66, l. I.), all angular errors are as the ſquare of the time of the moon's revolution directly, and the ſquare of the time of the earth's revolution inverſely; that is (by pr. 15, l. I.), as the ſquare of the time of the moon's revolution directly, and the cube of the earth's diſtance from the ſun inverſely.

[Pr. 30. And this force, by prop. 25, is, &c.] The force 3PK : force ML : : 3IT : PT (for theſe are the ſame), and force ML : centripetal force by which the moon revolves, &c. : : 1 : $178\frac{29}{40}$ (by pr. 25); therefore, *ex equo*, force 3PK : centripetal force the moon revolves with : : 3IT × 1 : PT × $178\frac{29}{40}$; or as IT : rad. × 59,575.

[Ib. the half of which the moon] it ſhould be, which the moon, by the action of the ſaid force, as it is in the firſt edition.

[Ib. And the angle PTM (Fig. 57) is equal to the angle, &c.] for in this caſe LM is perpendicular to MP. Let PR be a tangent to the point P; then the triangle RPT is a right one; and angle RPM = angle PTM. And angle LPM : ∠ (RPM =) PTM (when the radius is PM) : : LM : RM : : force producing LM : force producing RM : : 1 : 59,575.

[Pr. 31, cor. and the decrement is to the remaining motion as 100, &c.] The motion of the nodes in the octants : motion in the ſyzygies : : $11073^2 : 11023^2$: : (becauſe 11073, 11023, 10973, are in arithmetical, and nearly in ∺ progreſſion) 11073 : 10973; and decrement : remaining motion : : (11073 — 10973 =) 100 : 10973.

[Ib. but the decrement] (Fig. 58) for the decrements are as the forces and times, or as the whole motions and times; that is, decrement at H (or increment at h) : decrement at A : : mot. at H, and time : mot. at A, and time : : (that is, by what went before) as mot. at H × $\overline{yy - \frac{1}{2}rr}$: mot. at A × $\overline{rr - \frac{1}{2}rr}$ (or mot. at A × $\frac{1}{2}rr$). This reaſoning being very obſcure, you will find the motion of the nodes clearly inveſtigated in (prop. 5, ſect. 6, and cors. of) my Aſtronomy.

[Pr. 32, it is drawn back again] that is, ſuppoſing the place of the node given.

[Pr. 32. Now the area of the ſemi-circle] For let NT = 1, AZ = y, TZ = x. Then $eZ = \frac{y^3}{9.0827 + yy}$, and Ae $(AZ - eZ) = \frac{9.0827y}{10.0827 + yy}$. But $yy = 1 - xx$, therefore $Ae = \frac{9.0827\sqrt{1 - xx}}{10.0827 - xx} = \frac{\overline{b-1}\sqrt{1-xx}}{b - xx} = \overline{b-1} \times \frac{1}{b} + \frac{xx}{bb} + \frac{x^4}{b^3}$, &c. × into $\sqrt{1 - xx}$, putting b = 10.08276, and the fluxion area $NAe = \overline{b-1} \times \frac{1}{b} + \frac{xx}{bb} + \frac{x^4}{b^3}$, &c. × $\dot{x}\sqrt{1 - xx}$; whoſe fluent (by form 17, Fluxions) $= \frac{1}{b} + \frac{1}{4bb} + \frac{1}{8b^3}$, &c. × $\overline{b-1}$. ϕ (putting ϕ = quadrant, or $\frac{NAn}{2}$) = .92435 ϕ, therefore half the area NFn = .07565ϕ. And the area ϕ (or quadrant) is to the area $\frac{NFn}{2}$, or the ſemicircle to the whole area NF$\bar{n}$, as 1 to .07565, or as 793 to 60.

[Prop. 33, will nearly agree with] let N = 19 49 3 55, p = periphery of NAn or DFB, E the equation, S = S.2NA.

Then NAZ repreſents the mean motion of the nodes for the arc NA, as N does for the whole circle, and NAZ is the true motion; and ATZ is the difference proportional to the equation. Divide by $\frac{1}{2}$r, and then $p : \frac{xy}{r} :: N : E$, or $p : \frac{2xy}{r} :: \frac{1}{2}N : E$, that is, $p : S :: \frac{1}{2}N : E$; whence if BF = 2NA, then (by conſtruction) AD : CD : : p : $\frac{1}{2}$N : : S : E. Suppoſe S an arc in DF, then 360 : $\frac{1}{2}$N : : degrees in S : E; but deg. in S (in the circle DF) : degrees in DG (in the circle DG) : : AD : CD : : (by conſtruction) 360 : $\frac{1}{2}$N in degrees : : degrees in S : E; whence the degrees in DG = E, or the angle DAG = equation.

[Ib. cor.] for in the ſyzygies the areas ANZ and eNZ vaniſh; and NA is 90° when they are in the quadratures; and when in the octants the ſine of BF = S.90 = 1; therefore,

33.3	1.58319
Rad.	10.
1	0
S. < A (1° 30')	8.41681

[Pr. 2, p. 229, and by the demonſtration] the angle ATN will be the diſtance from the node's true place, by conſtruction. Alſo the mean motion of the ſun in the time NA : mean motion of the node from the ſun in the ſame time : : (mean annual motion of the ſun : mean annual motion of the node from the ſun : : (by pr. 1) area of the ellipſis : area of the circle : :) TBN : TFN; but TBN was the mean motion of the ſun in the time NA; and therefore TFN is the mean motion of the node from the ſun in that time NA; and FTN the angle of the mean motion, or the diſtance of the ſun from the mean plane of the node.

[Ib. cor.] for (Fig. 59) let atn = nth = 45°, and fc $\perp$ to at; then fc is the ſine of fta, the Δs fcb and bth are ſimilar; therefore fc : fb : : bt : bh; and, by compoſition, fh : ct, or at : : bf :

fc; and, by inverſion, fc is to at, or kt : : fb : fh, or by : : kH : Hm, or tk + tH.

[Ib. but the ſine of] draw oz ⊥ to it, then oz is to fc in a ratio compounded of (or to fb or of) os to fg, and (the ſine of orz or trs to the ſine of fbc or tbg, that is, of) ts to tg; that is, in the compound ratio of os × ts to fg × tg, that is (becauſe rad. : coſine : : 2^{ce} ſine : ſine of the double arc, and therefore the rectangle of the ſine and coſine being as the ſine of the double arc), as ſine of 2otn to ſine of 2ftn, or radius.

[Ib. ſch.] for TS : (TS + SK =) TK : : 360° : (360° + 39°,6355 =) 399,6355 : : 9,0827646 : 10,0827646. Therefore TH : TK : : $\sqrt{9,0827646}$: $\sqrt{10,08, \&c.}$ And therefore TH : HK : : 18,6524761 : 1 : : TS : SH : : mean motion of the ſun : mean motion of the node = $19^\circ\ 18'\ 1''\ 23'''\frac{2}{3}$.

Laſtly, TK + TH (= 38,22428) : KH (= 1) : : rad. : ſine of 1° : 29′ : 57″ (by cor.).

[Pr. 34, cor. 2, with due regard] for whilſt p moves from Q to F, the ſum of the areas is comprehended by TH produced, FQ, and tangent to Q; but, in moving from F to q, the line Hp falls on the other ſide of the circumference, and the ſum of theſe areas is that comprehended by FT produced, Fq, and the tangent to q; and the difference of theſe areas is the ſemi-circle generated in the time of deſcribing QAq; and in the time of deſcribing the whole circumference the whole circle will be generated.

[Ib. cor. 4, that is, as the diameter] Suppoſe Qp to be the double diſtance of the moon from the quadratures; MK the ſine thereof. Then Kk : Mp : : MK : radius. And the ſum of all the Kk's, or the diameter : ſum of all the Mp's or QAq : : ſum of all the ſines : ſum of as many radii, or half as many diameters. And, therefore (multiplying the antecedents by $\frac{Pp}{PG}$, and the conſequents by 2), the ſum of all the ſines × $\frac{Pp}{PG}$: ſum of as many diameters : : diameter × $\frac{Pp}{PG}$: whole circumference.

[Pr. 35, by the ſame increments as the ſine of inclination doth, by cor. 3.] For if AEG be double the diſtance of the nodes from the quadratures, GED will be double the diſtance of the nodes from the ſun; and they both have the ſame ſine.

[Ib. ſchol.] Here are ſeveral particulars about the moon's motion barely laid down, many of them being taken only from obſervations.

[Ib. p. 235. The force of this action is greater] It is greater the nearer it is, and attracts the moon from the earth, and ſo dilates the orbit the more. But the farther off, the leſs force, and the leſs the moon is drawn from the earth. And at a greater diſtance ſhe moves ſlower; and at a leſs diſtance faſter.

[Ib. farther, I found that the apogee] by cor. 14, pr. 16. b. I. And if a be the ſun's mean diſtance, and a + x any other diſtance, then the motion will be as $\frac{1}{\overline{a+x}^3}$, or $\frac{1}{a^3} - \frac{3x}{a^4}$, or $\frac{1}{a^3} \times \overline{1 - \frac{3x}{a}}$, and the equations, as $\frac{x}{a}$, or as x.

The motion of the ſun, that is, the angular motion, is reciprocally as the ſquare of the diſtance.

[Ib. but if the motion,] let v be the velocity, m the mean motion of the ſun. Then if v be as $\frac{1}{\overline{a+x}^2}$ or $\frac{1}{aa} - \frac{2x}{a^3} = m - \frac{2x}{a}m$, then the equation is $\frac{2x}{a}m$. But if v be as $\frac{1}{\overline{a+x}^3}$ or $\frac{1}{a^3} - \frac{3x}{a^4} = m - \frac{3x}{a}m$, then the equation is $\frac{3x}{a}m$. And the firſt equation to the ſecond is as $\frac{2x}{a}m$ to $\frac{3x}{a}m$, or as 2 to 3. And hence the greateſt equation of the apogee and nodes (ſum of all the $\frac{3x}{a}$ m) : their mean motion (m) : : 2° 54′ 30″ : ſun's mean motion; the greateſt equations being as the mean motions.

[Ib. p. 236. By the theory of gravity I likewiſe found] For then the moon being nearer the ſun (in all points of its orbit taken together), the ſun's force muſt be greater.

[Ib. p. 237. By the ſame theory of gravity the action,] For when the ſun is in the line of the nodes, its whole force is exerted in moving the moon, and none in moving the plane of its orbit.

[Ib. By the ſame theory of gravity, the moon's] by cor. 8, pr. 66, b. I.

[Pr. 36, in other poſitions of the ſun,] by the reaſoning in cor. 19, pr. 66, l. I. depreſſion of the water at P : aſcent at P in the direction PH : : (LM : TM : :) PT : 3PK; and aſcent at P in direction PH : aſcent at P in direction TP : : (rad. : S.PTK : :) PT : PK; therefore *(ex equo)* depreſſion at P : perpendicular aſcent there : : ($PT^2 : 3PK^2 :: \frac{1}{3}PT^2 : PK^2$; but the force PT that depreſſes P is given, therefore the force to raiſe the waters at P is always as PK^2, that is, as the verſed ſine of 2PTC, or twice the ſun's altitude.

Alſo the effects of theſe forces (by cor. 14, pr. 66, l. I.) at different diſtances from the ſun are reciprocally as the cubes of theſe diſtances; and, therefore,

Cor. 1. The ſun raiſes the water under it to one *Paris* foot and $11\frac{1}{30}$ inches.

[Pr. 37, but becauſe of the reciprocation] ſee pr. 24.

[Ib. and the ſun's force in —] (Fig. 60) let A be the poſition of the moon, and ATP = angle the ſun and moon makes at the earth's centre; and let PT repreſent the force of the ſun, which divide into the forces PK, KT, PK, acting in the direction of the moon, increaſes her force; KT acting in a different direction, diminiſhes it; and therefore PK — KT (or the difference of the ſine and coſine of the angle ATP) is the abſolute increaſe of the moon's force; but this differs not much from the coſine of double the $\angle$ ATP (in an angle of 18°½ the difference = ,0310189), and in the arcs 0° 45° 90° they perfectly agree. Or thus:

Let qc = moon's force, ca = ſun's force (Fig 61). If the ſun is in n, let $\angle$ ace = 2 $\angle$ acn; then qe or qb = ſum of the forces, cb = ſun's force, in that poſition = coſ. of 2 $<$ acn (Fig. 62).

[ib. But farther; the force of the moon] for let PT be the whole force of the luminary in the place P (or in D), the other ſide of (the earth or) its parallel; the force DT = the two forces DK, KT. When the luminary is in D, the force to move the water in the direction TP is TK; and the difference of theſe, or (TP — TK =) PK, is the difference of the forces in the points P, D; or the abſolute force to move the ſea at the lat. CTP. But PK is $\propto$ ▭ ſine of ATP, or as ▭ coſ. CTP.

Then ſince L + S is the whole force when the moon is near the ſyzygies, and L — S in the quadratures, therefore 0,8570327 L muſt be ſubſtituted for L only in the quantity L — S, which repreſents her force in the quadratures.

[Ib. from whence we have] alſo 1,017522 L muſt be ſubſtituted for L in L + S, which repreſents her force in the ſyzygies; and 0,9830427 L for L in L — S, when ſhe is in the quadratures; for theſe repreſent her attractive forces in the ſyzygies and quadratures.

[Pr. 38, as the accelerative gravity] for if the diameters be given, the forces to raiſe the water will be $\propto$ abſolute forces; and if the abſolute forces are given (the ſpheres will be reduced into ſimilar ſpheroids, for), the perturbating forces will be as the diameters (for the perturbating force is $\propto$ PT, Fig. pr. 25). Therefore, if neither be given, the forces to raiſe the water in the moon and earth are in the compound ratio of the earth to the moon, and the moon's diameter to the earth's diameter, or as 1090 to 100; therefore the water riſes to $93\frac{3}{4}$ feet in the moon.

No notice is taken here of the tides being leſs by reaſon of the moon's motion round the earth, than if ſhe ſtood ſtill in a given poſition. Since the motion of the tides reaches 90° for every ebbing and flowing, it would ſeem that the ſaid motion could not be propagated through 90° to its greateſt height, in the time the moon ſtays in the meridian. But (by a computation from pr. 44, II.) it appears that water will oſcillate in a canal of 90° in length in $\frac{2}{3}$ of an hour; and therefore in that time the tides will be propagated to that diſtance, and riſe to their higheſt; but the poſition of the moon in that time is not ſenſibly altered; and therefore the tides would riſe no higher, though ſhe were always to ſtand in a given poſition. But this ſuppoſes that there is depth enough of ſea to ſupply water ſufficient for the purpoſe.

[Lem. 1 to pr. 39, to recede towards this ſide and that ſide] that is, all the parts of AC to act in a direction towards the ſun; the parts of CE directly from it.

[Lem. 2. The matter in the circumference of every circle IK] by this is meant the circular ring (in the plane of the circle IK) comprehended between the ſurfaces of the ſphere and ſpheroid.

[Lem. 3, about the ſame axis] he means about its diameter.

[Ib.] I. Let (Fig. 63) ABD be a ſpherical ſurface, and AEFD a circumſcribing cylindric ſurface, and their common thickneſs the infinitely ſmall line Bb or Aa. Divide AC into an infinite number of equal parts $= \text{in} = \dot{x}$, and erect the planes id, ne; then the ſurfaces of the ſphere and cylinder comprehended thereby are equal. Now, flux. mot. cyl. ſurface : flux. mot. ſph. ſurface : : (as their matter × velocity, and the matter is given, and therefore as) $CB \times \dot{x}$, or $id \times \dot{x} : uf \times \dot{x}$. And mot. cyl. ſurface : mot. ſph. ſurface : : as all the $CB \times \dot{x}$: to all the $uf \times \dot{x} :: AD^2$: area of the circle ABDG.

Again; let (Fig. 64) ABD be a ſphere, and AEFD its circumſcribing cylinder; ECA a cone, all revolving round AD. Divide BC into an infinite number of equal parts $= bd = \dot{x}$. Through the points draw ſpherical ſurfaces bgd concentric to ABD, and cylindric ſurfaces bf terminated at the line EC. Then (by what went before) mot. cyl. ſurface bdf: mot. ſph. ſurface bdg : : AD^2 : circle ABDG. Therefore mot. all cyl. ſurfaces, or the ſolid EBC : mot. all the ſph. ſurfaces, or the ſphere : in the ſame given ratio of AD^2 : to circle ABDG.

Farther; flux. mot. cyl. (Fig. 65) round its axis $\propto$ flux. mot. × velocity, that is, $\propto ef^2 \times \dot{x}$, or $x^2\dot{x}$. And the

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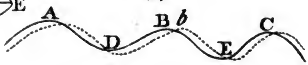

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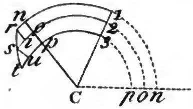

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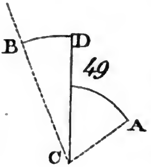

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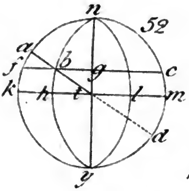

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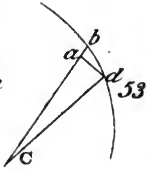

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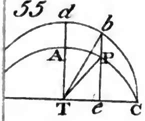

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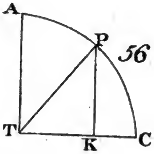

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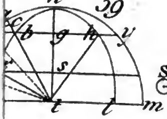

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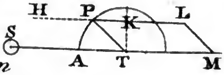

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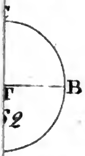

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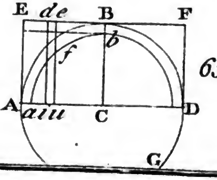


whole motion is $\propto \frac{x^3}{3}$. And therefore the motion of a cylinder round its axis $\propto$ length × cube of its diameter × denſity; or as its weight × diameter.

Laſtly (Fig. 66); let EBCA be a cylind. and ECA a cone. Then flux. cyl. mot. : flux. con. mot. : : $AE^3 \times \dot{x} : ba^3 \times \dot{x} :: CA^3 \times \dot{x} : Cb^3 \times \dot{x}$, or as $r^3\dot{x} : x^3\dot{x}$. Mot. cyl. round its axis : mot. cone round its axis : : $r^3x : \frac{x^4}{4}$: : (when $x = r = CA$) 4 : 1. Therefore mot. cyl. : mot. fig. EBC (= difference of mot. of cylinder and cone) : : 4 : (4 — 1 =) 3. Hence it follows,

Mot. cyl. : mot. ſol. EBC : : 4 : 3
Mot. ſol. EBC : mot. ſphere : : diameter $\square$: circle,
Therefore, *ex equo*,
Mot. cylinder : mot. of ſphere : : 4 diam. $\square$: 3 circle.
Or,
Mot. ſphere : mot. cylinder : : 3 circles : 4 diam. ſquare.

II. Flux. mot. cyl. : flux. mot. of a ring (or periphery) (Fig. 67) of the ſame diameter and matter : : (as velocity × flux. matter, as) $x^2\dot{x} : rx\dot{x}$. And mot. cyl. : mot. ring : : $\frac{x^3}{3} : \frac{rx^2}{2}$: : (when $x = r$) 2 : 3. And mot. cyl. : mot. any ring : : 2 cylinders : 3 rings.

III. Let (Fig. 68) $ED = z$, and let $\dot{z}$ be given; then $y\dot{z} = r\dot{x}$. Now flux. mot. ring round C : flux. mot. ring round AB : : $r\dot{z} : y\dot{z}$ or $r\dot{x}$. Therefore mot. ring round C : mot. ring round AB : $rz : rx$: : (when $x = r$) AD : AC : : circumference : 2 diameters.

Therefore from the 1ſt, 2d, 3d, art. it follows, *ex equo*, that the motion of a ſphere : motion of a ring round their axes : : as the matter in the cylinder and ring, and 6 × circumference × circle : 24 diam. cube : : or as ſquare of the quadrantal arc : to ſquare of the diameter : : ; that is, as matter in the cylinder to matter in the ring, and 61685 : to 100000; or as the matter in the ſphere to the matter in the ring, and 925275 to 1000000.

[Pr. 39. (for the reaſons above explained),] for (by cor. 2, prop. 30) mean horary motion of the nodes : $6'' \; 35''' \; 16'''' \; 36^{v}$: : $AZ^2 : AT^2$. And the ſum of all the mean horary motions throughout the year : ſum of as many $16'' \; 35'''$, &c. : : ſum of all the AZ^2 : ſum of as many AT^2 : : (by lem. 1, prop. 39) 1 : 2; for the diurnal rotation and annual motion of the earth are ſuppoſed uniform.

[Ib. But becauſe the plane of the equator,] For, by prop. 30, the diſturbing force is as 3PK (and in that prop., the inclination of the planes being ſmall, is not taken notice of); but when the plane of the orbit of P is inclined to the ecliptic, the line PK is projected into pK, and the force will therefore be diminiſhed in the ratio of PK to pK, or as the radius to the coſine of inclination; for the difference of the diſtances of P and T from the ſun become leſs in that proportion, and the difference of the forces are diminiſhed in the ſame ratio.

This being a propoſition of great difficulty, depending upon very intricate calculations, which few people are judges of, for this reaſon, the author's demonſtrations have been objected to, and cenſured by ſome people as not true. And Mr. Simpſon (in his Miſcellaneous Tracts), mentioning ſome of theſe cavillers, falls into the ſame notion. He has invented a ſort of motion (which he calls *Momentum*) unknown to Sir Iſaac Newton, or any body elſe, and which differs from Sir Iſaac's in the ratio of 800000 to 925725. But I never before heard of any motion that was not made up of the quantity of matter and velocity.

Another frivolous objection he makes, is, about the motion of a ring being different from that of the equator; and he tells us, that the motion *(Momentum)* of a ring round its diameter is only half of what it would be when revolving in its plane round the centre. But it is more than half, for it is as 1 to $\frac{3.1416}{2}$, as is demonſtrated in art. 3. What he writes afterwards (though he ſays *it is evident*) is not intelligible. But he concludes at laſt, in his way, that Sir Iſaac Newton has made the preceſſion (by the ſun's force) to be but half of what it ſhould be; and as ſome he mentions had made the whole propoſition erroneous, he modeſtly aſcribes but *two miſtakes* to Sir Iſaac in this one propoſition. But, I believe, that whoever reads the foregoing notes will ſoon be convinced that his demonſtrations are all right; and that all thoſe blunders they tell us of are entirely of their own making, and muſt be aſcribed to themſelves only.

[Lem. 4, p. 254. Wherefore if both the quantity of light,] for quantity of light is as the quantity the body receives from the ſun directly, and the ſquare of the diſtance from the body reciprocally; that is, $\propto$ ſquare of the apparent diameter directly, and ſquare of the body's diſtance from us reciprocally: therefore the body's diſtance is as the apparent diameter directly, and $\sqrt{\text{light}}$ reciprocally.

[Pr. 40, cor. 4] For then the parabola and earth's orbit touch in the vertex, and the area there will be $= \frac{1}{2}$ velocity $\times$ diſtance, or radius, or $\frac{1}{4}$ parameter; that is, $= \frac{10000 \times 243.2747}{2}$, and $\frac{10000 \times 10.1364\frac{1}{2}}{2}$ for the diurnal and horary motion. The reſt is plain, from pr. 14, l. I.

[Lem. 5.] (Fig. 69.) For the ordinates AH, BI, CK, &c. put a, 2a, 3a, &c. and take the differences as follow:

$$\begin{array}{ccccccccccc}
a & & 2a & & 3a & & 4a & & 5a & & 6a \\
 & b & & 2b & & 3b & & 4b & & 5b & \\
 & & c & & 2c & & 3c & & 4c & & \\
 & & & d & & 2d & & 3d & & & \\
 & & & & e & & 2e & & & & \\
 & & & & & f. & & & & &
\end{array}$$

Now to find any ordinate, as 4a, we have $4a = 3a - 3b$

$$= \begin{matrix} 2a - 2b \\ -2b + 2c \end{matrix} = \begin{matrix} a - b \\ -2 \times b + 2 \times c \\ + c \end{matrix}$$

$+ d = a - 3.b + 3.c - d.$

And ſo of others.

CASE 1. If we ſubſtitute for p, q, r, &c. their equals, we ſhall have,

$$\begin{aligned}RS &= a\\ &+ b\times -SH\\ &+ c\times -SH\times -\frac{SI}{2}\\ &+ d\times -SH\times -\frac{SI}{2}\times\frac{SK}{3}\\ &+ e\times -SH\times -\frac{SI}{2}\times\frac{SK}{3}\times\frac{SL}{4}\\ &+ f\times -SH\times -\frac{SI}{2}\times\frac{SK}{3}\times\frac{SL}{4}\times\frac{SM}{5}, \text{ \&c.}\end{aligned}$$

Now let RS fall upon any ordinate, as, for example, on LD; then SH, SI, SK, SL, &c. will be = LH, LI, LK, o, &c. = 3, 2, 1, o, &c. reſpectively; which being written in the aforeſaid ſeries, we have

$$RS = a - 3.b - 3\times\frac{-2}{2}c - 3\times\frac{-2}{2}\times\frac{-1d}{3} +$$

$o\,e = a - 3.b + 3.c - d$, which, by what went before, is the value of the ordinate DL; and ſince this holds generally, it is plain this ſeries will give the value of RS wherever it falls.

CASE 2. Let x repreſent any baſe HS, y any ordinate RS. Aſſume $y = A + Bx + Cx\times\overline{x-P} + Dx\times\overline{x-P}, \times\overline{x-Q}$, continued to as many terms as there are ordinates, which ſuppoſe four; P, Q, R, &c. being reſpectively = HI, HK, HL. Then, putting AH, BI, CK, DL, ſucceſſively for y and o, HI, HK, HL, for x, we ſhall have from the general equation $AH = A$, $BI = A + BP$, $CK = A + BQ + CQ\times\overline{Q-P}$, $DL = A + BR + CR\times\overline{R-P} + DR\times\overline{R-P}\times\overline{R-Q}$, that is, from the figure,

$AH = A = a.$

$BI = A + B\times HI.$

$CK = A + B\times HK + C\times HK\times IK.$

$DL = A + B\times HL + C\times HL\times IL + D\times HL\times IL\times KL.$

Whence, by Subtraction,

$AH - BI = -B\times HI.$

$BI - CK = -B\times IK - C\times HK\times IK.$

$CK - DL = -B \times KL - C \times \overline{HK + IL} \times KL - D \times HL \times IL \times KL.$

Then, dividing by the coefficients of B,

$\frac{AH - BI}{HI} = -B = b.$

$\frac{BI - CK}{IK} = -B - C \times HK = 2b.$

$\frac{CK - DL}{KL} = -B - C \times \overline{HK + IL} - D \times HL \times IL = 3b.$

Again, by Subtraction,

$b - 2b = C \times HK.$

$2b - 3b = C \times IL + D \times HL \times IL.$ And, dividing,

$\frac{b - 2b}{HK} = C = c.$

$\frac{2b - 3b}{IL} = C + D \times HL = 2c.$ Alſo

$c - 2c = -D \times HL$; and

$\frac{c - 2c}{HL} = -D = d.$

Thus the coefficients A, B, C, &c. are determined for four ordinates, and they are found the ſame way, if more ordinates are given. Then taking x at pleaſure, as ſuppoſe = HS, then RS or y will be $a + bx + cx \times \overline{x - P} + dx \times \overline{x - P} \times \overline{x - Q}$, &c. $= a + b \times p + c \times p \times IS + d \times p \times IS \times SK$, &c. $= a + bp + cq + dr$, &c.

[Cor. lem. 8, for then AC : Aδ (= AI — δI) : : AμC :] Aδdy — dμx (= AIμy — δIμX), and : : ASC : ASδ (Propor. 10) : : (AμC + ASC =) ASCμA : (Aδdy — dμx + ASδ = ASδdyA — dμx = (becauſe the Δs xδμ and δSμ are equal) ASδdyA — dμx + xδμ — Sδμ = ASδμdyA — Sδμ = ASμdyA : : time of deſcribing AμC : time of deſcribing Ayμ.

[Ib. Schol.] For Bn will cut the chord AC in a point a little beyond E (towards C), ſuppoſe at e. Now the area AEXμA to the area ACYμA is in ſomething a greater ratio than AE to AC, as ſuppoſe as Ae to AC, more near than before. Therefore ASEXμA : ASCYμA : : Ae : AC. Whence

$ASBX\mu A$ (or $ASEX\mu A$) : $ASCY\mu A$: : Ae : AC ; or $ASBY\mu A$: $ASCY\mu A$: : (Ae : AC : :) time in AB : time in AC, very near.

[Lem. 10, to the triangle ASC, that is,] $AC\mu A$: ASC : : $AC \times \frac{2}{3}M\mu$: $AC \times \frac{1}{2}SM$ (becauſe the <s SMA and $AI\mu$ are equal, and $M\mu = I\mu$) : : $AC \times 1\frac{1}{3}M\mu$: $AC \times SM$: : MN : SM. And, by compoſition, $ASC\mu A$: ASC : : SN : SM.

[Lem 11. Subduplicate proportion of 1 to 2,] and therefore $= \frac{AC}{\sqrt{2}}$. And the arc in half the time $= \frac{AI}{\sqrt{2}}$. And $\frac{AI^2}{2} \div 2SP = \frac{AI^2}{4SP} =$ ſpace deſcribed in deſcending.

[Prop. 41, and BE, by lem. 11, is a portion,] tS and this hypothenuſe (mentioned in the prop.) are the diſtances of the earth and comet from the ſun. Now BE to tV is compounded of BE to the part of the hypothenuſe (projected into BE), or of BS to the whole hypothenuſe, and of that part of the hypothenuſe to tV; that is, as the gravitating forces at the comet and earth (for theſe lines would be deſcribed, by falling bodies in equal times, by theſe forces); that is, as St^2 to hypothenuſe ſquare: and therefore BE : tV : : $BS \times St^2$: hypothenuſe cube.

Farther (Fig. 70); becauſe of the immenſe diſtance of S, any of the points B, i, μ, are nearly in the curve of the parabola, and μ the vertex, and μp, $i\lambda$, $q\sigma$, nearly parallel. Therefore $< I\mu i =$ (by reaſon of the rectangle $Ii\mu\lambda$) $< p\mu L$, or $= S\mu L$. Therefore μI is the diameter to the vertex μ; alſo $\xi\sigma =$ (by conſtruction) $3S\sigma + 3i\lambda = 3S\sigma + 3I\mu = 3S\sigma + 3\mu p =$ (becauſe $2O\sigma = 3\mu p$) $3S\sigma + 2O\sigma$; and therefore $\xi O = 3S\sigma + 3O\sigma = 3OS$. But μO : $I\mu$: : $p\sigma$: Ip. But μI is biſected by $i\lambda$; therefore $Ip = 2I\lambda$; and $p\sigma = I\lambda = \frac{1}{2}Ip$ (ſince $I\sigma = 3I\lambda$, by conſtruction); therefore $\mu O = \frac{1}{2}\mu I$. Whence (by lem. 8) a line drawn from ξ divides the chord AC nearly as the time; whence BE drawn towards ξ is rightly drawn. But (by conſtruction) new BE : former BE : : BS^2 : $\overline{S\mu + \frac{1}{3}i\lambda}|^2$: : as the gravitating force at the diſtance $S\mu + \frac{1}{3}i\lambda$: to gravitating force at the diſtance B or μ : : ſpace fallen through at the diſtance $S\mu + \frac{1}{3}i\lambda$, in the time of deſcribing the arc $\frac{1}{2}A\mu C$: ſpace fallen through at B or μ, in the ſame time. But former BE equal ſpace fallen through at μ. Therefore new BE $\doteq$ ſpace fallen through at the diſtance $S\mu + \frac{1}{3}i\lambda$, in half the time the comet deſcribes the arc intercepted between TA, $\tau C \doteq$ (by lem. 11) $i\lambda$, of this new BE, nearly; which, therefore, is rightly determined nearer than before.

Becauſe the diſtance of the ſun and comet is ſomething more than the hypothenuſe of the triangle, whoſe baſe is $S\mu$, and perpendicular IO (becauſe O is only a point in the chord, which ſhould be in the arc), therefore he ſuppoſes that diſtance = hypothenuſe (whoſe baſe is $S\mu + \frac{2}{3}i\lambda$, and perpendicular IO, that is) = DO. Alſo MP : X : : (by conſtruction) $\surd\overline{St}$: $\surd OD$: : (by cor. 6, pr. 16, I.) velocity of the comet at the diſtance DO : velocity thereof at diſtance St : : ſpace deſcribed (uniformly) at the diſtance DO, and in the time of deſcribing the arc ABC : ſpace deſcribed (uniformly) at the diſtance St, in the ſame time, which is equal X, by conſtruction; and therefore MP = ſpace deſcribed (uniformly) at the diſtance DO (in the time that the arc ABC is deſcribed) = (by lem. 10, cor.) to the chord of that arc.

The points e, a, c, g; and ε, α, χ, γ, being found out as E, A, C, G, before, is with an intent to find, at laſt, MP = MN, or AC = AG. Wherefore ſeveral of them being thus found, and a ſimple curve (viz. a circle) drawn through theſe points, finds the points Z and X, where thoſe lines would be equal; for the nearer they approximate to equality, the nearer they come to the true points of the comet's orbit.

Farther; velocity of a comet at Q (in a parabola) : velocity of a comet at Q (in a circle) (by cor. 7, prop. 16, I.) : : $\surd 2 : 1$: : (nearly as) $\frac{4}{3} : 1$. Alſo velocity at Q (in a circle) : velocity at t (in a circle) (by cor. 6, pr. 4, I) : : $\surd St : \surd SQ$. Therefore, *ex equo*, velocity of the comet at Q, in a parabola : velocity of the earth in its orbit at t : : $\frac{4}{3}\surd St : \surd SQ$. But velocity of the comet at Q : velocity of the earth at t : : (nearly as) AC : $T\tau$. Therefore AC : $T\tau$: : $\frac{4}{3}\surd St : \surd SQ$; or

AC : $\frac{4}{3}\mathrm{T}\tau$: : $\surd$St : $\surd$SQ. As it is by conſtruction (for AC is to $\frac{4}{3}\mathrm{T}\tau$ in the reciprocal ſubduplicate ratio of SQ to St, and not in the direct ſubduplicate ratio, as is falſely printed); therefore Q is (nearly) in the chord of the parabola, and B a point of the comet's orbit, nearly.

Laſtly; if MP = MN, or AG = AC; then Yb : YB : : Yc : YE : : ac : AC : : velocity in b : velocity in B : : $\surd$SB : $\surd$Sb. But if Sb = SB and MP or AG invariable, it will be Yb : YB : : as ac or AG : AC, when the point G falls in CY. Therefore, univerſally, Yb : YB : : AG × $\surd$SB : AC × $\surd$Sb : : MP × $\surd$SB : MN × $\surd$Sb, to find the point b true.

[Ib. p. 277, may be ſeen in the following table.] The places of the comet in this and all the other tables are the geocentric longitudes and latitudes of the comet.

[Ib. p. 283. And thinking, &c.] In Plate 18, while the comet paſſes through I, K, L, M, &c. the earth paſſes from between P and K towards G. Whence the comet will move ſwift through LKL and NOP; ſlower through LBM and QRS, as is ſaid in the foregoing p. 282.

[Ib. p. 285, as about 6 to 1000,] for, by p. 274-5, $\frac{\text{lat. rect.}}{4} =$ 6, nearly; when earth's diſt. = 1000.

[Ib. p. 290, I (Fig. 71) found, that at the height, &c.] Let SA = rad. earth = 20950000 Engliſh feet, nearly, AB = 850, AF = AS, and the denſities AH and BI are as 33 to 32. But (by cor. 2, p. 22, II.) Aa — Bb : Aa — Ff : : thin : thnz; that is, AB × SF, or 2AB × SA : AF × SB, or SB × SA : : thin : thnz; or 2AB : SB : : thin : thnz; that is, 1 : 12324 : : thin : thnz : : (by ſchol. pr. 86, hyperbola) log. St — log. Su : log. St — log. Sz : : ,013364 : 1.518514 — log. Sz. Whence log. Sz = — 164.820578; and SZ or FN = (,0^{163} ciphers) 66 +. Wherefore FN : AH : : ,0^{163} 66 : 33 : :, or as 2 : 10^{165} : : 1 : 10^{164} : : denſity in F : denſity in A. But the diameter of Saturn's orb is = 191 times the ſemidiameter of the orb's magnus = 191 × 20000 radii of the earth = 191 × 20000 × 20950000 × 12 inches = 96 0^{13} + inches. And a ſphere of 1 inch diameter : to this ſphere : : as 1 : $\overline{900^{13}}|^{3}$: : 1 : 88 0^{13} +; but 10^{164} > 8 0^{14}. Therefore, &c.

[Pr. 42.] the correction of the orbit here given is computed by the Rule of Falſe; 1ſt, for the longitude of the node, by operation 1 and 2; and, 2dly, for the inclination of the orbit, by operation 1 and 3.

I. for the longitude of the node; ſince (by conſtruction) time of deſcribing D : time of deſcribing E : : D : E : : G : 1; and (time of deſcribing E, nearly =) time B : time A : : 1 : C; therefore *(ex equo)* time of deſcribing D : time A : : G : C; therefore G — C is the error of the times between the firſt and ſecond obſervations, for the given longitude of the node K, and the given inclination of the orbit I. Again;

Time of deſcribing d : time of deſcribing e : : d : e : : g : 1; and time of deſcribing e (nearly =) time B : A : : 1 : C; therefore time of deſcribing d : time A : : g : C; and g — C is the error of time between the two firſt obſervations, for the longitude K + P, and inclination I; therefore, by the Rule of Falſe, by two poſitions (which ſays, diff. er. : diff. poſitions : : either er. : correction of its poſition), G — g : G — C : : P : $\frac{G - C}{G - g}$ P = the correction of K; or the computation may be thus: ſince to the ſuppoſition K and K + P the errors between the firſt and third obſervations in theſe two caſes are T — S, and t — S, therefore T — t : T — S : : P : $\frac{T - S}{T - t}$ P = the ſame correction of K; therefore mP (is to be) $= \frac{T - S}{T - t} P = \frac{G - C}{G - g}$ P; and therefore mT — mt = T — S; and mG — mg = G — C.

II. For the inclination of the orbit, it is plain T — S = firſt error of the times between the firſt and third obſervations to the given node K, and inclination I; and r — S = error between the firſt and third obſervations to the node K, and inclination I + Q. Therefore (by the Rule of Falſe) T — τ : T — S : : Q : $\frac{T - S}{T - \tau}$ Q = correction of I. Again; this time of deſcribing δ : time of deſcribing ε : : δ : ε : : γ : 1; and (time of deſcribing ε = nearly) time B : A : : 1 : C : *ergo* time of deſcribing δ : A : : γ : C; therefore γ — C = error of the times between the firſt and ſecond obſervations to the node K and inclination I + Q; and G — C was the error between the firſt and ſecond obſervations to the node K and inclination I; therefore (as before) $G - \gamma : G - C :: Q : \frac{G-C}{G-\gamma} Q$ = correction of I; therefore nQ (muſt be) $= \frac{T-S}{T-\tau} Q = \frac{G-C}{G-\gamma} Q$; and $nT - n\tau = T - S$, and $nG - n\gamma = G - C$. Laſtly; from (I. and II.) $2T - 2S = mT - mt + nT - n\tau$; and $2G - 2C = mG - mg + nG - n\gamma$; therefore the numbers m, n, are rightly found.

[Ib. and, laſtly, if in —] becauſe the longitudes in the firſt and ſecond orbits are K and K + mP, therefore the *latus rectums* will be R and $R + \overline{r - R} \times m$; and becauſe, in the firſt and third orbits or planes thereof, the inclinations are I and I + nQ, therefore the *latus rectums* will be R and $R + \overline{\varrho - R} \times n$; or $R + \overline{r - R} \times m$, and $R + \overline{r - R} \times m + \overline{\varrho - R} \times n$.

Or thus; if the longitude K be increaſed (in op. 1, 2) by P, then (becauſe the diſtance of the perihelion from the ſun is increaſed accordingly) the *latus rectum* will be increaſed by $r - R$; and if the longitude be increaſed by mP, the *latus rectum* will be increaſed by $\overline{mr - R}$. And, for the ſame reaſon (in op. 1, 3), if the inclination be increaſed to nQ, the *latus rectum* will be increaſed to $\overline{N\varrho - R}$ (for at the increaſe Q the increaſe of the *latus rectum* is $= \varrho - R$, ſince the whole is ϱ, or $R + \overline{\varrho - R}$); therefore if both longitude and inclination be increaſed, the *latus rectum* will be increaſed in both theſe reſpects, and will become $R + mr - mR + n\varrho - nR$.

Alſo when K becomes K + P, the tranſverſe becomes $\frac{1}{L + \overline{l - L}}$ or $\frac{1}{l}$; alſo when K becomes K + mP, the tranſverſe becomes $\frac{1}{L + \overline{ml - L}}$: for the ſame reaſon, when I is increaſed to I + nQ, the tranſverſe is $\frac{1}{L + \overline{n\lambda - L}}$.

4.

Therefore when both longitude and inclination are increaſed, the tranſverſe will then be $\frac{1}{L + ml - mL + n\lambda - nL}$.

[Ib. ſch. p. 307, the comet] for, by p. 285, comet's diſt. : earth's diſt from ⊙ : : 6 : 1000 : : diſt. comet in ⊙ diameters : (earth's diſt. in ⊙ diam. =) 109 ⊙ diam.; therefore comet's diſt. from the ſun's centre $= \frac{654}{1000} =$ 654 diams. and from its ſurface = ,154 diams. $= \frac{1}{6}$ ſun's diameter.

End of the Comment.

A
DEFENCE
OF
SIR ISAAC NEWTON,
AGAINST THE OBJECTIONS THAT HAVE BEEN MADE
TO
Several Parts of the Principia.

> Raro antecedentem ſceleſtum,
> Deſeruit pede pœna claudo. HOR. LIB. III.

THIS incomparable Treatiſe being written in a conciſe ſtyle, and in the ſynthetic method, and being upon ſubjects quite new and untouched before, the generality of readers could make little of it. As it contained a new ſyſtem of philoſophy, built upon the moſt ſublime geometry, the greateſt mathematicians were obliged to ſtudy it with great care and attention, and few became maſters of the ſubject; ſo that for a long time it was little read. But, at laſt, when the value of it became more known, it gained univerſal approbation, and the whole world ſtood amazed at the numberleſs new diſcoveries contained therein. And, upon account of its univerſal agreement with all the phænomena of nature, it was adopted as the only true ſyſtem by all, except ſome few that, through envy or ignorance, were bigoted to ſome other ſcheme.

It is true, many things therein depend upon very difficult mathematical computations not eaſily comprehended by every reader; and this has given occaſion to ſome people to queſtion, or even deny, the truth of ſeveral propoſitions laid down in this work; and have therefore given computations of their own, different from his, which they aſſert for truth, and thereby, as they think, proving his to be falſe. I ſhall, therefore, ſpend a little time in anſwering the moſt material of theſe objections, and ſhall ſhew that theſe principles objected againſt are all true in the ſenſe Sir Iſaac meant them ; and that theſe objections are only the effect of their own ignorance or inattention.

J. Bernoulli (tom. IV. p. 340) thinks Newton's demonſtration of cor. 4th to the laws is imperfect, where he ſays, *only the action of two bodies is conſidered*; and lays down a method by which he thinks it may be fully demonſtrated; but at laſt leaves it undemonſtrated himſelf, and tells you that this is the foundation, but leaves the calculation for others to make out.

But Sir Iſaac Newton, after he had ſhewn the truth of the cor. in two bodies, ſhews that the common centre of gravity of theſe two bodies and a third will either be at reſt, or move uniformly in a right line; and from thence that the centre of gravity of *any two* (in a ſyſtem of bodies) ſuffers no change of ſtate by their actions upon one another. From whence it follows, that in the whole ſyſtem all theſe actions (compoſed of the action of *every two)* can induce no change; and if the actions of *any two* make no difference in the centre of gravity of the whole, ſeparately conſidered, then the actions of *every two* conſidered collectively can make no difference. This is certainly ſo, though delivered in a few words: Sir Iſaac was not writing to children.

The ſame *Bernoulli* (p. 347) objects againſt the 10th pr. b. II. for not being ſolved in a way general enough to pleaſe him. Here Sir Iſaac has ſolved the problem according to that law of reſiſtance which really exiſts; but could not ſpare time, or fill up his work with uſeleſs enquiries, or things of no conſequence. His noble ſtructure, the *Principia*, is all gold; but he has left the droſs for ſuch authors as theſe, who are fond of any thing, and will ſerve well enough for building up their ſyſtems. This author in particular is always depreciating Sir Iſaac, and extolling his own performances; though they are long, tedious, and laborious to the laſt degree, and often falſe; although he has the advantage of the analytical and fluxional methods, and ſeems to be ignorant that the method of analyſis by which he *(Bernoulli)* ſolves theſe problems is incomparably ſhorter than the method of compoſition in which Sir Iſaac has written. Had he written the Principia in the analytic way, or according to the method of Fluxions, a ſcience then utterly unknown to the world, it could not have been read by any man living. By the ſame way of reaſoning, this boaſting mathematical bully may as well condemn *Euclid's* Elements, or *Apollonius's* Conics, becauſe moſt of the propoſitions may be demonſtrated incomparably ſhorter by common algebra.

This Author ſeems to have a particular ſpite againſt the Engliſh mathematicians, being always carping and criticiſing. His works are full of invectives againſt Sir Iſaac Newton for his great diſcoveries, which this low critic is unſucceſsfully endeavouring to imitate. When one perſon has another man's works to look through, he muſt be a great blunderbuſs that cannot make ſome ſmall additions: a man placed upon another's ſhoulders will ſee farther than his ſupporter; yet in many caſes he has diſimproved Sir Iſaac Newton. Sir Iſaac had a work entirely new to execute, in a ſhort and general way, and could not take notice of every bauble: this author has had nothing to do but to imitate. The regular works of the one will live through all ages; but the confuſed chaos of the other will ſink into everlaſting oblivion.

The ſame author (p. 484) alſo objects againſt the demonſtration of the 36th prop. b. II. For, ſays he, *ſince in Newton's cataract there is no compreſſion of the water in any place, nor againſt the ſides of the cataract, the external water preſſing inwards muſt diſturb the cataract, and mix with it; and, therefore, the Newtonian explication being contrary to the laws of Hydroſtatics, cannot ſubſiſt.* Yet what Newton ſays (caſe 1) might have ſatisfied him. His words are; *For let the ice in the veſſel diſſolve into water, yet will the efflux of the water, as to its velocity, remain the ſame as before. It will not be leſs, becauſe the ice, now diſſolved, will endeavour to deſcend: it will not be greater, becauſe the ice, now become water, cannot deſcend without hindering the deſcent of other water equal to its own deſcent.* But this author is eternally cavilling at every thing; and all his whole ſection on this ſubject, conſiſting of about 100 pages, where he has toſſed this problem about into all forms, is nothing but a heap of abſurd, inconſiſtent ſtuff, neither agreeable to theory nor experiment. So we ſhall leave him here drowned in a *gurges* of his own contriving.

In the demonſtration of prop, 47, b. II. Sir Iſaac Newton aſſumes for a principle, that the parts of the air have a motion excited in them, by ſome cauſe or other, according to the laws of an oſcillating pendulum. But *Euler* has found fault with him for making ſuch an hypotheſis; and has himſelf aſſumed ſome different hypotheſes, from which he has pretended to demonſtrate that the motion of the particles of air are moved according to ſuch hypotheſes; and that his demonſtration for his hypotheſis is as valid as Newton's is for his. But Sir Iſaac Newton's hypotheſis is more than an aſſumed one, for it is true in fact; for any tremulous or vibrating body from which ſound is propagated is known to vibrate according to this law, aſſumed by Newton; and the like motion is communicated to the particles of air. Therefore he has rightly aſſumed that the medium has ſuch a motion excited in it; and conſequently that it will continue to move according to that law. But there is no ſuch law of a vibrating body as *Euler* aſſumes, and therefore his demonſtrations come to nothing. To find the velocity of ſound is a very ſubtle problem; and they that would ſee it truly reſolved may find it in my book of Fluxions, prob. 23, ſect. 3.

In b. II. prop. 52, Sir Iſaac Newton has inveſtigated the properties of vortices, in order to ſee whether the celeſtial motions can be accounted for thereby; and he has ſhewn that the celeſtial bodies are not carried round in vortices. But *J. Bernoulli,* a perſon of eternal contradiction, has undertaken to defend the notion of vortexes, though ſhewn to be abſurd and impoſſible. Let him frame what hypotheſis he will for the planets, he muſt find vortexes meeting one another for the comets, that move all manner of ways. Therefore, to clear this point, he muſt have his vortexes made of ſuch fluid matter as has neither friction nor reſiſtance, and that one part be penetrable to another part; that is, ſuch a fluid as cannot be found in the world. Such is the blinded, bigoted, prejudice of ſome people, that the cleareſt demonſtrations cannot cure their madneſs. If any body aſks one of this ſtamp how the planets move round, they preſently anſwer, in vortexes. But aſk them what cauſes the vortexes to move round, here they are at a ſtand, and have nothing to ſay, but that they move by their own nature: but they might as well have ſaid at firſt that the planets move round by their own nature, if they can give no better account of them, and ſo ſave the labour of conſtructing theſe uſeleſs vortices.

Another author that is *vortex-mad*, is *Euler*, and he ſeems to go beyond any of the reſt, for he cannot account for the riſing of the tides without vortexes; but he has not ſhewn us by what extraordinary mechaniſm, or inviſible wheelwork, his vortices are conſtituted, ſo as to be able to produce the tides, or cauſe the motion of the comets, ſince theſe vortices muſt all run counter to one another, and penetrate one another, and yet miraculouſly preſerve their motions entire.

This idle notion was firſt introduced in the time of ignorance, upon the ſuppoſition of a *fuga vacui*, or that nature abhorred a *vacuum*; but, by our better acquaintance with the nature and properties of body, and the laws of motion, we now know that the operations of nature cannot be performed in a *plenum*, and therefore a *vacuum* is abſolutely neceſſary.

If we had known of no celeſtial bodies but the planets moving all one way, the ſuppoſition of a vortex to carry them about, clumſy as it is, might have paſſed for poſſible; but one would have thought that the comets' moving all manner of ways would have cured this delirious notion, and have taught them the impoſſibility of ſuch a ſcheme: but theſe authors, theſe defenders of vortices, are ſo hardy, that they are not at all afraid of an abſurdity or a contradiction. Theſe things do not affect or touch them in the leaſt; but they go on unconcerned in their uſual way, though contrary to all the laws of nature.

Did I ſay that the theory of vortices had its original from the principle of a *fuga vacui*? I did. And the principle of a *fuga vacui* had its riſe from the phænomena of water riſing in pumps and ſiphons, as in the *Torricellian experiment?* Here, for want of trying ſuch experiments to their full extent, the principle of a *fuga vacui* was aſſumed by theſe philoſophers as the genuine cauſe of theſe effects; ſo that, upon no better a footing than this, though the true cauſe of theſe effects (the external preſſure of the air) be now perfectly known, yet with theſe bigoted people the ſame falſe principle ſtill exiſts.

Theſe philoſophers (if they can be called ſuch) are endeavouring, contrary to the nature of things, to find out the moſt complicate cauſes for explaining the phænomena, inſtead of ſeeking the ſimpleſt cauſes; and they ſeem utterly to reject all ſimple cauſes, which are the greateſt beauty of nature; for if their brains were not turned round in a vortex, they could never prefer theſe complex vortical ſchemes before the ſimple doctrine of projectile and centripetal forces. Such Philoſophers!

Sir Iſaac Newton had ſhewn, in the ſchol. of prop. 14, b. III. that the aphelions of the interior planets move a little *in conſequentia* (by the actions of Jupiter and Saturn) in the ſeſquiplicate ratio of their diſtances from the ſun; but *Bernoulli* ſays this is falſe, for it holds not true in Saturn; and yet Sir Iſaac tells him expreſsly it is the inferior planets that obſerve that law. So little do ſome men care what they write, if they can only put on an air of contradiction.

In finding the proportion of the axis and diameter of the earth, prop. 19, b. III. Sir Iſaac aſſumes them to be as 100 to 101, and from thence finds the exceſs of weight in one above the other to be $\frac{4}{505}$ parts. Therefore he makes this proportion $\frac{4}{505} : \frac{1}{100} :: $ the force $\frac{1}{289} : \frac{1}{229}$, the true exceſs of height. Yet ſome people have objected againſt this as a wrong way. But herein they are much miſtaken; for if any force be increaſed, the effect will be increaſed in the ſame ratio; and therefore increaſing the centrifugal force here will increaſe the difference of the heights in the very ſame proportion, as theſe quantities are very ſmall. Upon this account, ſome people have unneceſſarily run into long calculations, and only came, at laſt, to the ſame concluſion.

And the ſame argument holds good againſt ſuch as have objected againſt finding the height of the tide (cor. prop. 36, b. III.) by the rule of proportion, comparing it with the centrifugal force; for all forces of whatever kind will produce *effects* proportional to the *quantities* of theſe forces; and therefore he has rightly found the height of the tide by that proportion.

But Sir Iſaac Newton's explanation of the tides (prop. 24, b. III.) does not pleaſe *Euler*, though he accounts for every circumſtance thereof. He thinks aſcribing theſe effects to the actions of the ſun and moon is recurring to *occult cauſes*, and therefore he had rather recur to *vortexes* for the explanation thereof, the notion of which has been confuted over and over. He denies the gravitation of bodies towards one another, becauſe he cannot diſcover the cauſe of gravity; and therefore he will not allow it to have any thing to do with the matter, as being an occult quality. But he recurs to a principle that is more than occult, his incomprehenſible vortices, which he thinks the tides are raiſed by, though he has not attempted to explain in what manner his vortices can do it.

He ſays, Sir Iſaac Newton's account of the tides is not ſufficiently explained, that any certain judgment can be formed whether it is true or ſalſe; that he did not explain the phænomena of the tides, but only darkened them. But certainly this man does not *know* what he writes, or does not *care* what he writes. Sir Iſaac explained all the principal phænomena thereof, and has ſhewn that his theory is agreeable to obſervations both in nature and quantity; and all matters of leſs note may be eaſily deduced from his general theory by any intelligent perſon. But Sir Iſaac did not chooſe to erect any *pontes aſinorum*.

This author values himſelf very much for rejecting the principle of *attraction* eſtabliſhed by *ſome Engliſhmen*, becauſe it is an occult quality. This profound Philoſopher will either know every thing or nothing; he will make no uſe of the forces of gravity, becauſe he knows not the cauſe of gravity. For the very ſame reaſon he would want to know the cauſe of that cauſe; and ſo he muſt know every thing in the whole chain of cauſes and effects, or he cannot be ſatisfied. This odd temper in ſome men ariſes from the pride of the human mind, which attempts to ſoar above its ſphere. They diſdain to know the little matter that is within their power to know; whilſt they are continually aſpiring at things that are without the reach of their knowledge; things for which they have no *faculties* ſuited to underſtand, and no *data* to determine. This is not to philoſophize, but to trifle.

As this Gentleman takes the liberty to ſneer at the *Engliſh*, he may pleaſe to ſtay and take this obſervation along with him,--that, if it had not been for an *Engliſhman*, he (and ſuch like) had known nothing. To him they are beholden for all theſe great and wonderful diſcoveries which all the world acknowledge and praiſe him for, except theſe ſons of detraction, envy, and ingratitude; diſcoveries that may be looked upon as the works of a genius rather divine than human.

The *Engliſh* have found, by experience, that, by the power of ſome cauſe or other, all bodies are drawn or impelled towards one another according to certain laws; and theſe laws are found out by obſervations, and a juſt reaſoning from the laws of motion. This cauſe, be it what it will, they call *gravity*, without pretending to determine of what kind it is. If this author knew the phyſical cauſe of gravity ever ſo well, he would be no wiſer; for the effects of it would be juſt the ſame; and we can meaſure the effects of gravity, and from thence find the quantities and proportions of the generating forces, without knowing what theſe forces conſiſt in; ſo that the knowledge of ſuch a cauſe would be only a uſeleſs ſpeculation. But this author is ſo fearful of occult cauſes, that he dare not make uſe of their manifeſt effects; and condemns the Engliſh at once for believing their own eyes that there are ſuch effects flowing from this unknown cauſe, which they have agreed to call *gravity*; for it appears to them that the effects thereof are all that they have any thing to do with. Sir Iſaac Newton tells him, more than once, *that he does not take upon him to define the kind or manner of action, or the cauſes or phyſical reaſon thereof;* but this author cannot take it in.

Some perſons, when they can find nothing elſe to ſay, have cavilled about the term *gravity,* and ſo their objections are dwindled to a diſpute about words, and not things. I ſuppoſe every thing in nature, that we have occaſion to make the leaſt uſe of in our diſcourſe and reaſoning, be it known or unknown, ought to have a name given it to diſtinguiſh it from other things, and to convey our meaning to others; and thus the *cauſe* of the acceleration of bodies towards one another is expreſſed by the word *gravity,* and the action itſelf is called *gravitation.* The word *attraction* is uſed in the very ſame ſenſe; that is, not in a *phyſical* but *mathematical* ſenſe, which regards only the *quantity* of the cauſe. That theſe terms are the moſt proper that could be choſen, will appear from hence: the word *attraction* is taken from its moſt manifeſt quality; for a body moving towards another with an accelerated motion has all the appearance poſſible of being drawn towards that other by ſome inherent virtue in the other. The word *gravity* has evermore expreſſed the tendency of a heavy body to the earth; and, therefore, by parity of reaſon, will as properly expreſs the tendency of the moon towards the earth, or of the earth towards the ſun, or of any body towards another body.

Suppoſe this philoſopher, or any other of that ſort, was aſked to calculate the times, ſpaces, or velocities of falling bodies (which is the moſt ſimple caſe that can be propoſed about gravity), would not any body juſtly laugh at him, if he ſtood to demur about it, and refuſe to calculate till he knew whether their falling was cauſed by attraction, impulſion, or the rotation of a vortex? And would not he equally deſerve to be laughed at, that ſhould heſitate to calculate the motions of the moon, or of the earth and planets, or of the tides, upon the ſame account, when they are all acted upon by the ſame unknown cauſe of gravity?

But, to return to the tides. This gentleman *(Euler)* tells us, that Newton's method is erroneous, by which he found the ſea to riſe to the height of near two feet, by the ſun's force only; and ſays, that Newton found out this enormous effect by comparing the ſun's force with the centrifugal force of the earth. This has been anſwered before; and certainly this Gentleman knows little about the nature of forces, if he does not allow that two equal forces, of however different kinds, will always have equal effects; and proportional forces proportional effects, eſpecially in their naſcent ſtate; for it is not the *kind* but the *quantity* of force that is to be regarded: therefore Newton rightly found the ſolar tide near two feet, and the lunar tide 8½ feet, agreeable to experience. But, to ſhew you what ſort of a theory this Gentleman works by, he finds the ſolar tide only half a foot, and the lunar tide 2¼ feet; in all not three feet; which all obſervations confute, and, with it, his erroneous method of computation. I have met with nobody yet but what makes it at leaſt three times as much.

He alſo tells us that Newton found out the forces of the ſun and moon by help of the tides; but he has not done it accurately: and yet Newton took in every circumſtance that could any way affect it, as may be ſeen in prop. 37, b. III.

Having had occaſion to compare different kinds of forces with one another, I will venture to lay down this as a general rule,---that all forces whatever, whether attractive or impulſive, centripetal or centrifugal, or of what kind ſoever, if they be equal, they will produce equal effects; and, therefore, how idle muſt it be for theſe men to wrangle about the *kind*, when the *quantity* only is concerned in the effect, and can only be of any real uſe to us in our calculations! The enquiry after the *kind* and *modus* of action is a phyſical or rather a metaphyſical ſpeculation, the knowledge of which they can never come at.

It has likewiſe been objected, by ſome perſons, that the two examples of Newton for finding the tides are ill choſen; but, however, he had no more to chooſe from, and by their near agreement it ſhews they were well choſen. *Euler* tells you, that, at *Havre de Grace*, the greateſt and leaſt tides are as

17 to 11; and therefore the ſun's force to the moon's will be as 17 — 11 to 17 + 11, or as 6 to 28; or, as he makes it, as 7·13 to 28, which is about as 1 to 4; a proportion not very different from Newton's. *Dan. Bernoulli* ſays, that, at St. *Malo*, the greateſt height to the leaſt is as 50 to 15, which makes the ſun's force to the moon's as 35 to 65, or as 7 to 13, not ſo much as 1 to 2; a concluſion utterly inconſiſtent with all other obſervations; which argues that the obſervation has not been made with ſufficient accuracy. However, this is certain,---that, if any place can be improper for ſuch an experiment, this place is, by reaſon of the very extraordinary tides; for here the tide, being hurried up a long channel growing continually ſtraighter, is forced up to an unuſual height.

However, I cannot think that it ſignifies a great deal where or in what places theſe experiments are made, provided the ſea be deep, and have free acceſs and receſs to and from the place of obſervation; for though the tides be higher in one place than another, the ſun and moon conſpire alike to that: for if the water be accelerated in any degree by the moon's force, it will likewiſe be accelerated proportionably by the ſun's force, ſo that the reſult will be nearly the ſame.

I cannot but be ſurpriſed, that, in ſo material a point as this, no perſon has been ſent purpoſely to proper places to make obſervations of the tides; ſince by this method only the forces of the ſun and moon can be determined to any tolerable degree of exactneſs. No celeſtial obſervations can aſſiſt us in this matter: aſtronomy affords us no help; and there ſeems to be no way for us to gain this great point, but this method by the tides. The forces of the ſun and moon are ſo very ſmall, in reſpect to the force of gravity, that no common hydroſtatical experiment can ſhew us the leaſt effects thereof: it is only in the tides that their effects become ſenſible.

The forces of the ſun and moon reach to the very centre of the earth, and act upon every point of the radius or column of water under them, and diminiſh the gravity of every particle thereof; and all theſe forces conſpire together to raiſe the tide;

and therefore the radius of the earth becomes the proper ſcale for all theſe forces to act on. And we cannot ſee the total effect of all theſe forces, unleſs we have a depth equal to the whole radius of the earth, or an extent of ſea of 90 degrees; and this effect on the tides, when at the greateſt, will then only amount to a few feet.

If I was to give directions for making obſervations on the tides for this purpoſe, I would adviſe to chooſe ſome place near the equinoctial; as, on the coaſts of *Africa* or *America*: and the place would be moſt advantageous where the ſun and moon are in the zenith at the firſt obſervation; and in the horizon at the ſecond, for the ſpring tides; and one in the zenith, and the other in the horizon, for two obſervations at the neap tides: and ſuch places ſhould be choſen where the ſea is of large extent, and deep, ſo as to communicate freely with the place, backward and forward. Such places will be beſt in a calm country, and performed in calm weather; and all circumſtances of weather ſhould be alike (as near as can be) at any two correſpondent times of obſervation---the ſyzygies and quadratures. Theſe obſervations ſhould be made at the ſyzygies and quadratures, both day and night; and alſo when the tides are higheſt and loweſt, which is three days after. But, for ſome purpoſes, trial ſhould be made every day; and the buſineſs of trying at the ſyzygies and quadratures ſhould be continued for a twelvemonth: and more places than one ſhould be tried. Perhaps ſome iſland in the middle of the ſea, as *St. Helena,* may be proper; for the weather is likely to be more uniform than on the continent.---Having thus gained a ſufficient number of obſervations, the beſt may be ſelected, and a mean ratio found, by which this matter will be finally determined.

There are ſome people that object to this method of finding the ſun's and moon's forces by the tides; and reckon it very precarious, and ſubject to many obſtacles and intervening cauſes, by which the tides are perpetually influenced and diſturbed; as if every thing had not its difficulties. The only diſturbing cauſe is the wind: yet they can tell us of no other method, but what is more precarious, more impracticable, and leſs exact. So much of the tides.

The 39th prop. is about finding the preceſſion of the equinoxes: againſt this, being a problem of great difficulty, objections have been raiſed by ſeveral people; alleging, that it is not truly demonſtrated. Mr. *Simpſon* is among this clan; and he abſurdly makes the preceſſion by the ſun's force alone to be 21″ 6‴, which is double to Sir Iſaac Newton's: the conſequence of which is, that the motion by the moon's force will be only 29″; ſo that, by this, the moon's force will be to the ſun's only as $1\frac{2}{5}$ to 1: yet he ſays, in another place, that the moon's force to the ſun's cannot be leſs than $2\frac{1}{2}$ to 1: ſo inconſiſtent and erroneous are his operations. But this has been taken notice of before in the Comment. But the moon's force muſt be greater than $2\frac{1}{2}$; for, in all obſervations of the tides that have been regularly made, the moon's force is 4, or more: and it is hardly poſſible, in any obſervation of the tides made with any tolerable degree of care, to miſs near a half. They that would ſee the preceſſion of the equinoxes truly calculated, will find it in prob. 26, ſect. 3, of my Fluxions.

Other objections, of leſs moment, I paſs over, as their malignity and falſehood will appear to every reader; ſuch as the abſurd opinion that motion cannot be loſt in the world (concerning which, ſee my Mechanics, 4to., prop. 10, and cors.); alſo the falſe opinions of thoſe that deny that the compoſition and reſolution of forces are analogous to the compoſition and reſolution of motions, which are their adequate effects; their endleſs cavils againſt his method of demonſtrating any propoſition, which is the ſynthetical, preferring their own analytical methods, as being ſhorter, which is no wonder; their introducing into phyſical calculations an obſcure, precarious law or force, called *vis viva,* founded upon no certain principles (concerning which, ſee my Mechanics, 4to., ſchol. to prop. 11, where the nature of it is unravelled); their promiſcuouſly uſing the words force and motion for one another, which are as different as cauſe and effect, which induces no ſmall obſcurity into their writings; their aſcribing the invention of *Fluxions* to *Leibnitz,* contrary to the cleareſt teſtimonies: and ſome of them tell us, that, *not he that firſt found, but he that firſt publiſhed it deſerves the praiſe;* as if the publiſher could have publiſhed it all, if the inventor had not firſt found it out. Here the inventor is robbed of his due praiſe, to reſign it to the thief that ſtole it: and, in general, their aggravating every trifling ſlip as a capital crime; and, inſtead of praiſing him for what he has done (which is more than all the world ever did before), they diſpraiſe him for what he *has not* done, or had not time to do; and lampoon him, becauſe his time or his knowledge was *not infinite.*

But if any body ſhould aſk what any of theſe *Bravos* have done ſince? the anſwer is this----Nothing at all, or leſs than nothing: they have been turning ſcience backwards; for they have been doing nothing but undermining his principles (though built upon the ſureſt foundations), to introduce their own chimerical hypotheſes, that have nothing to ſupport them but impudence, ignorance, and preſumption!

At the end of the Principia, Sir Iſaac Newton has given us his thoughts of the Deity. Here he ſhews that God is an *eternal, infinite, and powerful Being;* that all the frame of nature is owing to him, which he made, and governs: and, from the ſimilitude of all the parts of the world, he ſhews that God is *one;* that he is a Being acting with *council* and *deſign,* and with the greateſt *wiſdom;* that we have ideas only of his *attributes,* but know nothing of his *ſubſtance,* nor after what manner he acts or does any thing. Indeed, we know nothing of the ſubſtance of any thing, much leſs of God; and as we cannot conceive of ſpace and time, but as neceſſarily exiſting, much more muſt we allow that God exiſts neceſſarily; and, conſequently, that he exiſts *always,* and *every where,* and that not *partially,* but *totally.* In ſhort, he has laid down the beſt metaphyſical notions of God that can be met with any where.

But M. *Leibnitz* will not allow that God has in himſelf (much leſs other animals) any principle or power of acting, but as he is firſt acted on by ſome *motive,* which he thinks is to determine his actions; and, in conſequence of that, if two equal ways of acting were laid before God, he could chooſe neither, but be like a balance acted on by two equal weights, that would turn no way. But at this rate God is not a *free agent,* but a mere *patient.* He reckons it an *im-*

perfection in God to be able to chooſe *one* out of two perfectly equal things; becauſe, he ſays, there wants a *ſufficient reaſon,* and therefore he can chooſe neither. But it is certainly a *greater imperfection* to chooſe *neither* than to chooſe *one.* It is a principle with him that God muſt have a ſufficient reaſon; which is true : but it is God's mere will that is the ſufficient reaſon. And how comes he to know what is a ſufficient reaſon to a perfectly free agent that has it in his power to do any thing; as if man, who is no more than a *worm* in the creation, can preſume to know what is fitteſt to be done?

Sir Iſaac Newton has aſſerted, that all the great bodies in the world move in free ſpace, which is unbounded and infinite; that all bodies have pores or empty ſpaces within them. But this author will not allow any *vacuum,* but will have the world be a perfect *plenum*; and he tells us, alſo, that the world will continue for ever without any alteration (or mending): but he has taken particular care to prevent that by introducing his *plenum.* The motions of the heavenly bodies muſt needs be retarded, and ſoon ſtopt, in moving through denſe matter, though ever ſo fluid; ſo that one of his ſuppoſitions is inconſiſtent with another, and nobody could more effectually deſtroy his own hypotheſis than he has done himſelf. He had had infinitely better chance for this ſuppoſition if he had made the planets, &c. to move in a vacuum.

If God had deſigned every thing to ſtagnate, and be fixed in the world, then a plenum ſeems to be the beſt conſtitution for that end; but as all the operations of nature are to be performed by motion, which, indeed, is the beauty of nature, then placing them in vacuo muſt needs be the beſt conſtitution, as the motions will then be free and unreſiſted, and the moſt durable poſſible; and therefore it is as neceſſary to have a *vacuum* in which theſe motions are to be performed, as it is neceſſary to have *body* to perform theſe motions.

The ſame objections he has made againſt empty ſpace he makes againſt time; for with him ſpace is no more than the order of co-exiſting beings, and time the order of ſucceſſive ones; but at this rate the words *nearer* or *farther, ſooner* or *later,* ſignify nothing; yet both ſpace and time are meaſured by quantity, and therefore are themſelves quantities, and conſequently real beings, which confutes his notions. And, indeed, common ſenſe is enough to determine this; for their exiſtence is ſelf-evident, even to the moſt ignorant, and cannot be made more evident by all the arguments in the world. Arguments in ſuch caſes ſerve for nothing but perplexing things that are plain of themſelves, and are often brought for that purpoſe. *Actions* and a *vacuum* are the firſt principles of the moſt antient philoſophy; which is the ſame thing as to ſay, that God has made *bodies,* and *room* for them to act and move in; for the whole world appears to our ſenſes to be nothing but *matter* and *motion.*

This learned philoſopher's argument againſt empty ſpace is this: *every perfection which God could give to things, without derogating from their other perfections, he has given them. Suppoſe, then, an empty ſpace, God could have placed matter in it (which is more excellent than empty ſpace); therefore he has done it, and conſequently there is no vacuum.* Here he ſuppoſes that there is no excellency or perfection at all in motion; and ſo this world-maker will not agree to leave any room or free ſpace for bodies to move in; and in this he is as dogmatical as if he had been originally one of God Almighty's privy council. But, had he gone one ſtep farther, he would have ſeen the abſurdity of it; for God did not make the world to ſtand ſtill, but to move after various laws and rules; and, conſequently, when this acute philoſopher fills the planetary regions with matter, he *derogates from the perfections* of the other bodies by deſtroying their motion with his new matter; and in this caſe, and for this very purpoſe, empty ſpace is more excellent than matter.

But the *ignis fatuus* that leads him aſtray is the argument of *a ſufficient reaſon,* when, at the ſame time, he is no judge at all what is ſufficient. Yet this principle he brandiſhes about with no ſmall oſtentation and aſſurance: but it is of no more uſe in his hands than the ſword of *Achilles* in the hands of an infant.

End of the Defence.

THE

LAWS

OF

THE MOON'S MOTION

according to

GRAVITY.

BY JOHN MACHIN.

THE

LAWS

OF

THE MOON's MOTION.

IN juſtice to the editor of this tranſlation of Sir *Iſaac Newton's Principia,* it is proper to acquaint the reader, that it was with my conſent he publiſhed an advertiſement, at the end of a volume of miſcellanies, concerning a ſmall tract which I intended to add to his book by way of appendix; my deſign in which was to deliver ſome general elementary propoſitions, ſerving, as I thought, to explain and demonſtrate the truth of the rules in Sir *Iſaac Newton's* Theory of the Moon.

The occaſion of the undertaking was merely accidental; for he ſhewing me a paper which I communicated to the author, in the year 1717, relating to the motion of the nodes of the moon's orbit, I recollected that the method made uſe of in ſettling the equation for that motion was equally applicable to any other motion of revolution. And therefore I thought that it would not be unacceptable to a reader of the *Principia* to ſee the uſes of the ſaid method explained in the other equations of the moon's motion; eſpecially ſince the greateſt part of the Theory of the Moon is laid down without any proof; and ſince thoſe propoſitions relating to the moon's motion, which are demonſtrated in the *Principia,* do generally depend upon calculations very intricate and abſtruſe, the truth of which is not eaſily examined, even by thoſe that are moſt ſkilful; and which, however, might be eaſily deduced from other principles.

But in my progreſs in this deſign, happening to find ſeveral general propoſitions relating to the moon's motions which ſerve to determine many things which have hitherto been taken from the obſervations of aſtronomers; and having reaſon to think that the theory of the moon might by theſe means be made more perfect and complete than it is at preſent, I retarded the publication of the book, till I could procure due ſatisfaction by examining obſervations on places of the moon. But, finding this to be a work requiring a conſiderable time, not only in procuring ſuch places as are proper, but alſo in performing calculations, upon a new method, not yet accommodated to practice by convenient rules, or aſſiſted by tables, I thought it, therefore, more convenient for the bookſeller not to ſtop the publication of his impreſſion any longer upon this account. But that I may, in ſome meaſure, ſatisfy thoſe who are well converſant in Sir *Iſaac Newton's Principia* (and I could wiſh that none but ſuch would look over theſe papers) that the ſaid advertiſement was not without ſome foundation, and that I may remove any ſuſpicion that the deſign is entirely laid aſide, I have put together, although in no order, as being done upon a ſudden reſolution, ſome of the propoſitions, among many others, that I have by me, which ſeem chiefly to be wanting in a theory of the moon, as it is a ſpeculation founded on a phyſical cauſe; and thoſe are what relate to the ſtating of the mean motions. For although it be of little or no uſe in aſtronomy to know the rules for aſcertaining the mean motions of the node or apogee, ſince the fact is all that is wanting, and that is otherwiſe known by comparing the obſervations of former ages with thoſe of the preſent, yet in matter of ſpeculation this is the chief and moſt neceſſary thing required; ſince there is no other way to know that the cauſe is rightly aſſigned but by ſhewing that the motions are ſo much and no more than what they ought to be.

But that it may not be altogether without its uſe, I have added all the rules for the equation of the moon's motion, except two; one of which is a monthly equation of the variation depending on the moon's anomaly; and the other an equation ariſing from the earth's being not in the focus of the moon's orbit, as it has been ſuppoſed to be, in all the modern theories ſince *Horrox*.

For not having had time to examine over the obſervations which are neceſſary, but being obliged, inſtead thereof, to take Sir *Iſaac Newton*'s theory for my chief guide and direction, I cannot venture to depart from it too far in eſtabliſhing equations entirely new; ſince I am well aſſured, upon the beſt authority, that it is never found to err more than ſeven or eight minutes.

And, therefore, hoping that the reader, who conſiders the ſudden occaſion and neceſſity of my publiſhing theſe propoſitions at this time, will make due allowance for the want of order and method, and look upon them only as ſo many diſtinct rules and propoſitions not connected, I ſhall begin, without any other preface, with ſhewing the origin of that inequality which is called the Variation or Reflection of the Moon.

The variation of the moon.

The variation or reflection is that monthly inequality in the moon's motion, wherein it more manifeſtly differs from the laws of the motion of a planet in an elliptic orbit. *Tycho Brahe* makes this inequality to ariſe from a kind of libratory motion backwards and forwards, whereby the moon is accelerated and retarded by turns, moving ſwifter in the firſt and third quarter, and ſlower in the ſecond and fourth, which inequality is principally obſerved in the octants.

Sir *Iſaac Newton* accounts for the variation from the different force of gravity of the moon and earth to the ſun, ariſing from the different diſtances of the moon in its ſeveral aſpects.

The mean gravity of the moon to the ſun, he ſuppoſes, is ſatisfied by the annual motion of the moon round the ſun; the gravity of the moon to the earth, he ſuppoſes, is ſatisfied by a revolution of the moon about the earth. But the difference of the moon's gravity to the ſun, more or leſs than the earth's gravity, he ſuppoſes, produces two effects; for, as this difference of force may be reſolved into two forces, one acting in the way, or contrary to the way, of the moon about the earth, and the other acting in the line to or from the earth, the firſt cauſes the moon to deſcribe a larger or ſmaller area in the ſame time about the earth, according as it tends to accelerate or retard it; the other changes the form of the lunar orbit from what it ought to be merely from the moon's gravity to the earth; and both together make up that inequality which is called the variation.

But ſince the real motion of the moon, though a ſimple motion cauſed by a continual deflection from a ſtraight line by the joint force of its gravity to the ſun and earth, thereby deſcribing an orbit which incloſes not the earth but the ſun, is yet conſidered as a compound motion made from two motions, one about the ſun, and the other about the earth, becauſe two ſuch motions are requiſite to anſwer the two forces of its gravity, if ſeparately conſidered; for the very ſame reaſon, the moon's motion ought to be reſolved into a third motion of revolution, ſince there remains a third force to be ſatisfied, and that is the force ariſing from the alteration of the moon's gravity to the ſun. And this, when conſidered, will require a motion in a ſmall ellipſis in the manner here deſcribed.

The circle ADFH (Fig. 1) repreſents the orbit of the moon about the earth in the centre T, as it would be at a mean diſtance, ſuppoſing the moon had no gravity to any other body but the earth. The diameter ATF divides that part of the orbit which is towards the ſun, ſuppoſe ADF, from the part oppoſite to the ſun, ſuppoſe AHF. The diameter at right angles HTD is the line of the moon's conjunction with or oppoſition to the ſun. The figure PQLK is an ellipſis, whoſe centre is carried round the earth in the orbit ABDEFH, having its longer axis PL in length double of the ſhorter axis QK, and lying always parallel to TD, the line joining the centres of the earth and ſun. Whilſt the ſaid figure is carried from A to B, the moon revolves the contrary way from Q to N, ſo as to deſcribe equal areas in equal times about the centre of it; and to perform its revolution in the ſame time as the centre of the ſaid elliptic epicycle (if it may be ſo called) performs its revolution; the moon being always in the remoter extremity of its ſhorter axis in Q and K when it is in the quarters, and in the neareſt extremity of its longer axis at the time of the new and full moon.

The ſhorter ſemi-axis of this ellipſis AQ is to the diſtance of its centre from the earth AT in the duplicate proportion of the moon's periodical time about the earth to the ſun's periodical time: which proportion, if there be 2139 revolutions of the moon to the ſtars in 160 ſidereal years, is that of 47 to 8400.

The figure which is deſcribed by this compound motion of the moon in the elliptic epicycle, whilſt the centre of it is carried round the earth, very nearly repreſents the form of the lunar orbit; ſuppoſing it without eccentricity, and that the plane was coincident with the plane of the ecliptic, and that the ſun continued in the ſame place during the whole revolution of the moon about the earth.

From the above conſtruction it appears that the proportion between the mean diſtance of the moon and its greateſt or leaſt diſtances is eaſily aſſigned, being ſomething larger than that which is aſſigned by Sir *Iſaac Newton* in the 28th propoſition of his third book. But as the computation there given depends upon the ſolution of a biquadratic equation, affected with numeral co-efficients, which renders it impoſſible to compare the proportions with each other ſo as to ſee their agreement or diſagreement, except in a particular application to numbers, I ſhall therefore ſet down a rule, in general terms, derived from his method, which will be exact enough, unleſs the periods of the ſun and moon ſhould be much nearer equal than they are. Let L be the periodical time of the moon, S the period of the ſun, M the ſynodical period of the moon to the ſun, and D be the difference of the periods of the ſun and moon; then, according to Sir *Iſaac Newton*'s method, the difference of the two axes of the moon's elliptic orbit, as it is contracted by the action of the ſun, is to the ſum of the ſaid axes as $3L \times \frac{M + L}{2}$ to

4DD — SS. But, according to the conſtruction before laid down, the ſaid proportion is as 3LL to 2SS — LL.

By Sir *Iſaac Newton*'s rule, the difference will be to the ſum nearly as 5 to 694; and conſequently the diameters will be nearly as 689 to 699, or 69 to 70: but, by the latter rule, the difference will be to the ſum nearly as 1 to 119; and the diameters or diſtances of the moon, in its conjunction and quadrature with the ſun, will be as 59 to 60. Dr. *Halley* (who in his remarks upon the lunar theory, at the end of his catalogue of the ſouthern ſtars, firſt took notice of this contraction of the lunar orbit in the ſyzygies from the phenomena of the moon's motion) makes the difference of the diameters to the ſum as 1 to 90; and conſequently the greater axis to the leſſer as 45½ to 44½.

But the difference in theſe proportions of the extreme diſtances, though it may appear conſiderable, is not, however, to be diſtinguiſhed by the obſervations on the diameters of the moon, whilſt the variations of the diameters, from this cauſe, are intermixed with the other much greater variations ariſing from the eccentricity of the orbit.

The angle of the moon's elongation from the centre (Fig. 1), deſigned by BTN, is properly the variation or reflection of the moon; the properties of which are evident from the deſcription.

Firſt, It is as the ſine of the double diſtance of the moon from the quadrature or conjunction with the ſun; for it is the difference of the two angles BTA and NTA, whoſe tangents, by the conſtruction, are in a given proportion.

Secondly, The variation is, *cæteris paribus*, in the duplicate proportion of the ſynodical time of the moon's revolution to the ſun; for the variation is in proportion to the mean diameter of the epicycle, and that is in the duplicate proportion of the ſynodical time of revolution.

The greateſt variation is an angle whoſe ſine is to the radius as the difference of the greateſt and leaſt diſtances TQ and TL, that is, 3AQ, to their ſum. According to the proportion of the lines before deſcribed, this rule makes the elongation near 29 minutes; which would be the variation, ſuppoſing the moon performed its revolution to the ſun in the time of its revolution round the earth; but if that elongation of 29 minutes be increaſed in the duplicate proportion of the ſynodical time to the periodical time of revolution, it will produce near 34 minutes for the variation.

It is to be noted, that what is ſaid of the epicycle, is upon ſuppoſition that the earth's orbit round the ſun is a circle: if the eccentricity of the annual orbit be conſidered, the mean diameter of the epicycle muſt increaſe or diminiſh reciprocally in the triplicate proportion of the ſun's diſtance.

The method of finding the inequalities in any revolution.

The conſtruction which I communicated to Sir *Iſaac Newton* for the annual motion of the nodes of the moon's orbit (which is printed in the ſcholium to the 33d propoſition of his third book) is a caſe of a general method for ſhewing the inequality of any motion round a centre, when the hourly motion or velocity of the object varies according to any rule, depending on its aſpect to ſome other object; for in any revolution the mean motion and inequality are to be aſſigned by means of a curvilinear figure, wherein equal areas are deſcribed about the centre in equal times; the property of which figure is, that the rays from the centre are always reciprocally in the ſubduplicate proportion of the hourly motion or velocity about the centre.

Thus in the figure deſcribed in my conſtruction (Fig. 2), where TN is the line of the nodes, TA the line drawn to the ſun, is ſuppoſed to revolve round the centre T, with the velocity of the ſun's motion from the node; and the ray TB, which is taken always in the ſubduplicate proportion of that velocity, will deſcribe equal areas in equal times; ſo that the ſector NTB will be the mean motion of the ſun, the ſector NTA the motion of the ſun from the node, and conſequently the area NAB the motion of the node, which will be a retrograde motion if the area be within the circle, and direct if it falls without. From whence it follows,

1. That the periodical time of the ſun's revolution to the node will be to the periodical time of the ſun's revolution as the area of the curvilinear figure to the area of the circle:

2 That if a circle be deſcribed, whoſe area is equal to the area of the curvilinear figure, it will cut that figure in the place where the ſun has the mean motion from the node.

3. If an angle NTF be made, which ſhall comprehend an area in the ſaid circle, equal to the ſector NTB in the figure, that angle will be the mean motion of the ſun from the node. And, conſequently,

4. The angle FTB, which is the difference between the ſun's true motion from the node, deſigned by ATN, and the ſun's mean motion from the node, deſigned by FTN, will be the equation for the ſun's motion from the node, when the ſun's poſition to the node is deſigned by the angle ATN.

From all which it appears, that what is ſaid of the ſun's motion from the node will hold as to any other motion round a centre; as of the ſun from the moon, or the moon from the node or apogee. In any ſuch revolution a curvilinear figure may be deſcribed about the centre, by the areas of which the relation between the mean and true motion may be ſhewn; and, conſequently, the inequality or equation of the motion.

And as in every revolution there is a certain figure which is proper to ſhew this relation, ſuch a figure may be called an equant for that motion or revolution.

And in every revolution where the equant is a figure of the ſame property, the inequalities or equations will alter according to the ſame rule.

Thus, if the equant be an ellipſis about the centre, as in that for the motion of the ſun from the node,

Firſt, The mean motion in the whole revolution will be a geometrical mean proportional between the greateſt motion in the extremity of the leſſer axis and the leaſt motion in the extremity of the longer axis; for the radius of the circle, which is equal to an ellipſis, is a mean proportional between the two ſemi-axes.

Secondly, The tangents of the angles of the mean and true motion are in the given proportion of the two axes of the ellipſis. Thus the tangents of the angles of the true and mean motion of the ſun from the node, viz. the tangents of the angles ATN and FTN (Fig. 2), are in proportion as the ordinates BG and FG, that is, as the ſemi-axes TH and TN.

Thirdly, The ſine of the angle of the greateſt inequality in the octants is to the radius as half the ſum of the axes to half their difference.

It is to be noted, that the equant is an ellipſis about the centre in every motion where the exceſs of the velocity about the centre above the leaſt velocity is always in the duplicate proportion of the ſine of the angle of the true motion from the place where the velocity about the centre is leaſt. From which remark, upon examination, it will appear, that the following motions are to be reduced to an elliptic equant deſcribed about the centre.

The monthly motion of the moon from the node;

The annual motion of the ſun from the node;

The motion of the moon from the ſun as it is accelerated or retarded by the alteration of the area deſcribed about the earth, according to Sir *Iſaac Newton*'s 26th prop. 3d book;

And the annual motion of the ſun from the apogee. How theſe ſeveral equants are determined will appear by what follows.

The motion of the nodes.

The node is in its ſwifteſt retrograde motion when the ſun and moon are in conjunction or oppoſition, and in a quadrature with the line of the nodes. According to Sir *Iſaac Newton*'s method (explained at the end of the thirtieth propoſition of the third book), the force of the ſun to produce a motion in the node at this time is equal to three times the mean ſolar force; that is, by the conſtruction of the elliptic epicycle, equal to a force which is to the force of gravity as 3AQ to AT (Fig. 1), or three times the leſſer ſemi-axis of the ellipſis to the diſtance of its centre from the centre of the earth. But if the moon revolve in the elliptic epicycle as before deſcribed, the force to make a motion in the node at the time mentioned will be to the force of gravity as 3DL to DT, or three times the longer ſemi-axis to the diſtance of the centre; which is the double of the former force. But, then, according to Sir *Iſaac*'s method, the motion of the node at this time is to the moon's motion as the ſolar force to create a motion in the node is to the force of gravity. But if the moon be conceived as revolving in a circle, with the velocity of its motion from the node at this time, when the node moves ſwifteſt, and the plane of the ſaid circle be ſuppoſed to have a rotation upon an axis perpendicular to the plane of the ecliptic, and the contrary way to the motion of the moon, ſo as to produce the motion of the node, and leave the moon to move with its own motion about the earth, the force to make a motion in the node ſeems to be the difference of the forces to retain it with the velocity of its motion in the moveable and immoveable planes; but the velocities of bodies revolving in circles are in the ſubduplicate proportion of the central forces. From whence it follows, that

The motion of the moon from the node at this time, when the node moves ſwifteſt, is to the motion of the moon in the ſubduplicate proportion of the ſum of the forces to the force of gravity, or as the ſum of TD *and* 3DL *to* TD.

And this would be the greateſt motion of the node, upon ſuppoſition that the plane of the moon's orbit was almoſt coincident with the plane of the ecliptic; but if the inclination be conſidered, the motive force for the node muſt be diminiſhed in the proportion of the ſine-complement of the inclination to the radius. How much this motion is, will appear by the following ſhort calculation.

The diſtance TD being, as before, equal to 8400, and 3DL being 282, the inclination of the plane in this poſition is 4° 59′ 35″, the ſine-complement of which is to the radius as 525 to 527 nearly; therefore the force of gravity is to the motive force for the node thus diminiſhed in the compound proportion of 8400 to 282, and of 527 to 525, that is, in the proportion of 4216 to 141. So that the greateſt motion of the moon from the node is to the motion of the moon in the ſubduplicate proportion of 4357 to 4216, that is, in the proportion nearly of 613 to 603. According to which calculation the greateſt hourly motion of the node ought to be 32″ 47‴. By Sir *Iſaac Newton*'s method, it amounts to 33″ 10‴½.

This is the ſwifteſt retrograde motion of the node, when the line of the nodes is in a quadrature with the ſun, and the moon is in its greateſt latitude in conjunction or oppoſition to the ſun. But the equant for the motion of the moon from the node in this month, when the line of the nodes is in quadrature with the ſun, is an ellipſis about the centre; and therefore the mean motion in this month will be known by the following rule:

The mean motion of the moon from the node, in that month when the line of the nodes is in a quadrature with the ſun, is a geometrical mean proportional, between the greateſt motion of the moon from the node and the motion of the moon.

And therefore this mean motion will be to the motion of the moon in the ſubduplicate proportion of 613 to 603, that is, nearly in the proportion of 1221 to 1211. So that the mean motion of the node in this month will be to the motion of the moon as 10 to 1211, which makes the mean hourly motion 16″ 19‴$\frac{1}{40}$. According to Sir *Iſaac Newton*, it amounts to 16″ 35‴; but, by the corrections which he afterwards uſes, it is reduced to 16″ 16‴$\frac{1}{3}$.

But the equant for the annual motion of the ſun from the node being alſo an ellipſis, it follows, that

The mean motion of the ſun from the node is a geometrical mean proportional between the motion of the ſun and the mean motion of the ſun from the node in the month when the line of the nodes is in quadrature with the ſun.

How near this rule agrees with the obſervations, will appear by this calculation.

Since the mean motion of the node in that month when the line of nodes is in quadrature to the ſun was before ſhewn to be to the moon's mean motion as 10 to 1211, and the motion of the ſun is to the motion of the moon as 160 to 2139, it follows, that the motion of the node and the motion of the ſun will be in the proportion of 154 and 1395; and, therefore, by the rule, the ſun's mean motion from the node is to the ſun's mean motion in the ſubduplicate proportion of 1549 to 1395, that is, nearly as 98 to 93; which correſponds with the obſervations, there being 98 revolutions of the ſun to the node in 93 revolutions of the ſun The ſubduplicate proportion taken more nearly is as 941 to

893, which will produce 19° 21′ 3″ for the motion of the node from the fixed ſtars in a ſidereal year. The motion (as obſerved) is 19° 21′ 22″.

Had the calculation from the rule been more exactly made in large numbers, the annual motion produced would be 19° 21′ 07″½, which is 14″ leſs than the motion, as obſerved by the aſtronomers.

Which difference may very probably ariſe from the ſun's parallax; and, if ſo, it may perhaps furniſh the beſt and moſt certain method of adjuſting and fixing the true diſtance of the ſun. For the ſun's force being ſomething more on that half of the orb which is towards the ſun than what it is on the other half, the elliptic epicycle is accordingly larger in the firſt caſe than in the latter. And by calculation I find that the mean motion of the node, ariſing after conſideration is had of this difference, is more than the mean motion from the mean magnitude of the epicycle by near 2″ in the year, for every minute in the parallactic angle of the orbit of the moon, or for every ſecond of the ſun's parallax. And, by the beſt computation I have yet made, this difference of 14″ in the annual motion of the node will ariſe from about 8″ of parallax; which will make the ſun's diſtance above 25000 ſemi-diameters of the earth.

In like manner as the equant for the motion of the node in that month when the line of the nodes is in quadrature with the ſun is an ellipſis, ſo in any other month it is alſo an ellipſis; the motion of the node being direct and retrograde by turns, in the moon's paſſing from the quadrature to the ſun to the place of its node, and from the place of its node to the quadrature.

The inclination of the plane of the moon's orbit to the plane of the ecliptic.

But theſe elliptic equants do not only ſerve to ſhew the inequality of the motion of the node, but alſo the inclination of the plane of the moon's orbit to the plane of the ecliptic. Thus the rays in the elliptic equants, for the motion of the moon from the node in each month, deſign the inclinations of the plane of its orbit to the plane of the ecliptic in the ſeveral reſpective poſitions of the moon to the line of the nodes. And the rays of the elliptic equant for the annual motion of the ſun from the node, in my conſtruction (in the ſchol. to prop. 33, book 3, of Sir *Iſaac Newton's Principia),* deſign the different mean inclinations of the ſaid plane to the plane of the ecliptic in each month, when the ſun is in each reſpective aſpect to the line of the nodes.

Thus if NT (Fig. 2) (the ſemi-tranſverſe axis of the elliptic equant for the motion of the ſun from the node) deſign the mean inclination of the plane, or, which is the ſame thing, if it repreſent the mean diſtance between the pole of the ecliptic and the pole of the moon's orbit, in that month when the ſun is in the line of the nodes, TH, the ſemi-conjugate axis of the ſaid ellipſis, will deſign the mean inclination, or mean diſtance of the poles in that month when the line of nodes is in quadrature to the ſun; and TB, any other ſemi-diameter of the ſaid ellipſis, will repreſent the mean diſtance between the ſaid poles, when the ſun is in that aſpect to the line of nodes which is deſigned by the angle NTA. For example, if the leaſt inclination deſigned by the ſhort ſemi-axis TH be 5° 00′ 00″; ſince TH is to TK as the motion of the ſun to the mean motion of the ſun from the node, by the property of this equant; and ſince there are 98 revolutions of the ſun to the node in 93 revolutions of the ſun; it follows, that HK, the difference between the greateſt and leaſt of the mean inclinations in the ſeveral months of the year, is to TH, the leaſt, as 5 to 93; by which proportion the ſaid difference will amount to 16′ 10″. According to Sir *Iſaac Newton*'s computation, in the 35th prop. of the third book, it is 16′ 23″½. But if the ſaid number be leſſened in the proportion of 69 to 70, acccording to the author's note at the end of the 34th prop. the ſaid difference will become 16′ 9″.

And in like manner the inclinations of the plane of the moon's orbit, in that month when the motion of the node is ſwifteſt (being ſituated in the line of quadratures with the ſun), are determined by the equant for the motion of the moon from the node in that month.

Thus, let TH be to TN (Fig. 2) in the ſubduplicate proportion of the moon is motion to its greateſt motion from the node, when the moon is in the conjunction in TH; that is (as was before determined), let TH be to TN in the proportion of 1211 to 1221; and the ellipſis deſcribed on the ſemi-axes TH and TN will be the equant for the motion of the moon from the node in that month. And the rays of the ſaid equant will deſign the inclinations of the plane in the ſeveral aſpects of the moon to the line of the nodes; that is, if TN be the inclination of the plane, or the diſtance of the pole of the ecliptic from the pole of the moon's orbit, when the moon is in TN, the line of the nodes, the ray TB will repreſent the diſtance of the ſaid poles, or the inclination of the plane, in that aſpect which is deſigned by the angle NTB.

Which being laid down, it follows that the whole variation of the inclination, in the time the moon moves from the line of the nodes to its quadrature in THK, is to the leaſt inclination as KH to TH, that is, as 10 to 1211. Wherefore if the leaſt inclination be 4° 59′ 35″, the whole variation will be 2′ 29″. This is upon ſuppoſition that the ſun continued in the ſame poſition to the line of the nodes during the time that the moon moves from the node to its quadrature. But, the ſun's motion protracting the time of the moon's period to the ſun in the proportion of 13 to 12, the variation muſt be increaſed in the ſame proportion, and will therefore be 2′ 41″. According to Sir *Iſaac Newton's* computation, as delivered in the corollaries to the 34th prop. of the 3d book, for ſtating this greateſt variation (the intermediate variations in this or any other month not being computed or ſhewn by any method), it amounts to 2′ 43″. But if the ſaid quantity be diminiſhed in the proportion of 70 to 69, according to his note at the end of the ſaid propoſition, it will become the ſame preciſely as it is here derived from the equant.

The variation of the area deſcribed by the moon about the earth.

The motion of the moon from the ſun, as it is accelerated or retarded by the increment of the area deſcribed about the earth (according to the 26th prop. of the 3d book), is alſo to be reduced to an elliptic equant, by taking the ſhorter axis to the longer axis in the ſubquadruplicate proportion of the force of the moon's gravity to the earth to the ſaid force added to three times the mean ſolar force, that is, as TA (Fig. 1) to the firſt of three mean proportionals between TA and TA + 3AQ. And in the ſame proportion is the area deſcribed by the moon about the earth, when in quadrature with the ſun, to the mean area, or as the mean area to the area deſcribed in the ſyzygies: ſo that the greateſt area in the ſyzygies is to the leaſt in the quadratures in the ſubduplicate proportion of TA + 3AQ to TA, or as $\sqrt{8541}$ to $\sqrt{8400}$. This is upon ſuppoſition that the moon revolves to the ſun in the ſame time as it revolves about the earth; which will be found to agree very nearly with Sir *Iſaac Newton*'s computation in the before-cited propoſition.

The motion of the apogee.

And after the ſame manner an elliptic equant might be conſtructed, which would very nearly ſhew the mean motion of the apogee, according to the rules delivered by Sir *Iſaac Newton* (in the corollaries of the 45th prop. of the firſt book) for ſtating the motion of the apogee, namely, by taking the greateſt retrograde motion of the apogee from the force of the ſun upon the moon in the quarters, and the greateſt direct motion from the force of the ſun upon the moon when in the conjunction or oppoſition; each according to his rule, delivered in the ſecond corollary to the ſaid propoſition. And if an ellipſis be made whoſe axes are in the ſubduplicate proportion of the moon's motion from the apogee, when in the ſaid ſwifteſt direct and retrograde motions, the ſaid ellipſis will be nearly the equant for the motion of the moon from the apogee, and will be found to be nearly of the form of that above for the increment of the area.

But the motion of the apogee, according to this method, will be found to be no more than 1° 37′ 22″ in the revolution of the moon from apogee to apogee, which (according to the obſervations) ought to be 3° 4′ 7½″.

So that it ſeems there is more force neceſſary to account for the motion of the moon's apogee than what ariſes from the variation of the moon's gravity to the ſun, in its revolution about the earth.

But if the cauſe of this motion be ſuppoſed to ariſe from the variation of the moon's gravity to the earth, as it revolves round in the elliptic epicycle, this difference of force, which is nearly double the former, will be found to be ſufficient to account for the motion, but not with that exactneſs as ought to be expected. Neither is there any method that I have ever yet met with upon the commonly received principles which is perfectly ſufficient to explain the motion of the moon's apogee.

The rules which follow concerning the motion of the apogee, and the alteration of the eccentricity, are founded upon other principles, which I may have occaſion hereafter to explain, it being, as I apprehend, impoſſible to derive theſe, and many other ſuch propoſitions, from the laws of centripetal forces.

Let TC (Fig. 1) (in the above conſtruction of the lunar orbit) be the mean diſtance of the moon, or half the ſum of its greateſt and leaſt diſtances, viz. TQ and TL; and let CL be the mean ſemi-diameter of the elliptic epicycle, or half the ſum of the ſemi-axes; and take a diſtance LM, on the other ſide towards the centre, equal to CL; then

The mean motion of the moon from its apogee is to the mean motion of the moon in the ſubduplicate proportion of TM *to* TC.

For example; half the ſhorter axis or DC is 23½; therefore TC the mean diſtance is 8376½; CM or 2CL, the ſum of the ſemi-axes, is 141; ſo that TM is 8235½. Wherefore the motion of the moon from the apogee is to the motion of the moon in the ſubduplicate proportion of 8235½ to 8376½, or of 16471 to 16753, that is, nearly as 117 to 118, or more nearly as 352 to 355; or yet more nearly, as 1877 to 1893; ſo that there ought to be about 16 revolutions of the apogee in 1893 revolutions of the moon; which agrees to great preciſeneſs with the moſt modern numbers of aſtronomy, according to which proportion the mean motion of the apogee in a ſidereal year ought to be 40° 40′ 40½″. But, by the numbers in Sir *Iſaac Newton*'s theory of the moon, the ſaid motion is 40° 40′ 43″. According to the numbers of *Tycho Brahe,* it ought to be 40° 40′ 47″.

The mean motion of the apogee being ſtated, I find the following rule for the alteration of the eccentricity.

The variation of the eccentricity.

The leaſt eccentricity is to the mean eccentricity in the duplicate proportion of the ſun's mean motion from the apogee of the moon's orbit to the ſun's mean motion. Or in the duplicate proportion of the periodical time of the ſun's revolution to the mean periodical time of its revolution to the moon's apogee.

By the foregoing rule for the mean motion of the apogee there are 16 revolutions of the apogee in 1893 revolutions of the moon; but there being 254 revolutions of the moon in 19 revolutions of the ſun, there muſt be about 7 revolutions of the apogee in about 62 revolutions of the ſun, or rather about 20 in 177. So that the periods of the ſun to the ſtars; and of the ſun to the moon's apogee, are in proportion nearly as the numbers 157 and 177. The duplicate of which proportion is that of 107 to 136; which, according to the rule, ought to be the proportion of the leaſt eccentricity to the mean eccentricity.

So that by this rule the mean eccentricity (or half the ſum of the greateſt and leaſt) ought to be to the difference of the mean from the leaſt (or half the difference of the greateſt and the leaſt), as 136 to 29.

How near this agrees with the obſervations, will appear from the numbers of Mr. *Horrox,* or Mr. *Flamſted,* and of Sir *Iſaac Newton.*

The mean eccentricity, according to Mr. *Flamſted,* or Mr. *Horrox,* is 0.055236; half the difference between the greateſt and leaſt is 0.011617; which numbers are in the proportion of 135½ to 28½ nearly.

According to Sir *Iſaac Newton,* the mean eccentricity is 0.05505; half the difference of the greateſt and leaſt is 0.01173; which numbers are in proportion nearly as $135\frac{47}{48}$ to $28\frac{47}{48}$, each of which proportions is very near that above aſſigned.

But it is to be noted, that the rule which is here laid down is true only upon ſuppoſition that the eccentricity is exceedingly ſmall. There is another rule derived from a different method, which preſuppoſes the knowledge of the quantity of the mean eccentricity; and which will not only determine the variation of the eccentricity according to the laws of gravity with greater exactneſs, but ſerve alſo to correct an hypotheſis in the modern theories of the moon, in which their greateſt error ſeems to conſiſt; and that is, in placing the earth in the focus of that ellipſis which is deſcribed on the extreme diameters of the lunar orbit; whereas it ought to be in a certain point nearer the perigee, as I may have occaſion to explain more fully hereafter.

The equation of the apogee.

The greateſt and leaſt eccentricity being determined, the equant for the motion of the ſun from the apogee is an ellipſis whoſe greater and leſſer axes are the greateſt and leaſt eccentricities; and, therefore, by the property of ſuch an equant as before laid down,

The ſine of the greateſt equation of the apogee will be to the radius as the difference of the axes of the equant is to their ſum; that is, as the difference of the greateſt and leaſt eccentricities to their ſum.

For example; ſince the difference is to the ſum as 29 to 136, by what was determined in the foregoing article, the greateſt equation of the apogee will be about 12° 18′ 40″. Sir *Iſaac Newton* has determined it from the obſervations to be 12° 18′.

The greateſt and leaſt eccentricities being determined, the eccentricity and equation of the apogee, in any given aſpect of the ſun, are determined by the equant in the following manner.

Let TN be the greateſt eccentricity, TH (Fig. 2) the leaſt; the ellipſis on the ſemi-axes TN and TH the equant for the motion of the apogee.

Then if the angle NTF be made equal to the mean diſtance or mean motion of the ſun from the apogee, the angle NTB will be the true diſtance or motion of the ſun from the apogee; the difference BTF the equation of the apogee; and the ray TB the eccentricity of the orbit, in that aſpect of the ſun to the apogee deſigned by the angle NTB. Hence ariſes this rule:

The tangent of the mean diſtance, viz. NTF, *is to the tangent of the true diſtance* NTB *in the given proportion of the greateſt eccentricity* TN *to the leaſt* TH, *that is, as* 165 *to* 107.

From what has been laid down concerning the general property of an equant, that it is a curve line deſcribed about the centre, whoſe rays are reciprocally in the ſubduplicate proportion of the velocity at the centre, or the velocity of revolution, it will not be difficult to deſcribe the proper curve for any motion that is propoſed; and where the inequality of the motion throughout the revolution is but ſmall, there is no need of any nice or ſcrupulous exactneſs in the quadrature of the curve for ſhewing what the equation is. Thus all the ſmall annual equations of the moon's motion ariſing from the different diſtances of the ſun, at different times of the year, may be reduced to one rule exact enough for the purpoſe.

For ſince the ſun's force to create theſe annual alterations is reciprocally in the triplicate proportion of the diſtance, the rays of the equant for ſuch a motion will be in the ſeſquiplicate proportion of the diſtance. From whence it will not be difficult to prove, that, if the revolution of the motion to be equated were performed in the time of the ſun's revolution, the equation would be to the equation of the ſun's centre nearly as 3 to 2: and ſo if the force decreaſed as any other power of the ſun's diſtance, ſuppoſe that whoſe index is m, the equation would be to that of the ſun's centre as m to 2. But if the motion be performed in any other period, the equation will be more or leſs in the proportion of the period of the revolution to the ſun to the period of the revolution of the motion to be equated. Thus if it were the node or apogee of the moon's orbit, the equation is to the former as the period of the ſun to the node or apogee to the period of the node or apogee. Which rule makes the greateſt equation for the node about 8′ 56″, being a ſmall matter leſs than that in Sir *Iſaac Newton*'s theory; and the greateſt equation for the apogee about 21′ 57″, being ſomething larger than that in the ſame theory.

The like rule will ſerve for the annual equation of the moon's mean motion. If, inſtead of the equation for the ſun's centre, another ſmall equation be taken in proportion to it as the force, by Sir *Iſaac Newton* called the mean ſolar force, to the force of the moon's gravity, or as 47 to 8400, the ſaid equation increaſed in the proportion of the ſun's period to the mean ſynodical period of the moon to the ſun, or of 99 to 8, will be the annual equation of the moon's mean motion. According to this, the equation, when greateſt, will be 12′ 5″.

What is ſaid may be ſufficient for the preſent purpoſe, which is only to lay down the principal laws and rules of the ſeveral motions of the moon, according to gravity. Some other propoſitions, which ſeem no leſs neceſſary than the former for completing the theory of the moon's motion, as to its aſtronomical uſe, I reſerve to another time.

But, to make ſome amends for the ſhortneſs and confuſedneſs of the preceding propoſitions, I ſhall add one example to ſhew the uſe of the equant more at large, in what is commonly called the ſolution of the *Keplerian* problem; that being one of the things which I propoſed to explain, when the elements for the theory of the moon were advertiſed.

An Example of the Uſe of the Equant in finding the Equation of the Centre.

LET the figure ADP (Fig. 3) be the orbit in which a body revolves, deſcribing equal areas in equal times by lines drawn from a given point S; and let it be propoſed to find the equant for the apparent motion of the ſaid body about any other place within the orbit, ſuppoſe F.

Let there be a line FR indefinitely produced, which revolves with the body as it moves through the arc AR; and in the ſaid line take a diſtance Fp, which ſhall be to FR, the diſtance of the body from the given point F, in the ſubduplicate proportion of the perpendicular let fall upon the tangent of the orbit at R from the point S to the perpendicular on the ſaid tangent let fall from the given point F; and the curvilinear figure, deſcribed by the point p, ſo taken every where, will be the equant for the motion of the body about the point F.

For ſince the areas deſcribed at the diſtances Fp and FR are in the duplicate proportion of thoſe lines, that is, by the conſtruction, in the proportion of the perpendiculars on the tangents let fall from S and F, the areas which the body deſcribes in moving through the arc AR about the points S and F are in the proportion of the ſame perpendiculars; and therefore the area deſcribed by the revolution of the line Fp in the figure will be equal to that which is deſcribed by the revolution of the line SR in the orbit. So that the areas deſcribed in the figure will be equal in equal times, as they are in the orbit; and, conſequently, the rays Fp of the figure will conſtantly be in the ſubduplicate proportion of the velocity of the motion, as it appears at the centre F, which is the property of the equant.

From which conſtruction it will be eaſy to ſhew, that, in the caſe where a body deſcribes equal areas in equal times about a fixed point, there may be a place found out within the orbit, about which the body will appear to revolve with a motion more uniform than about any other place.

Thus, ſuppoſe the orbit ADP was a figure, wherein the remoteſt and neareſt apſes A and P were diametrically oppoſite in a line paſſing through the point S, viz. the point about which the equal areas are deſcribed; then if the point F be taken at the ſame diſtance from the remoteſt apſis A as the point S is from the neareſt apſis P, the ſaid centre F will be the place about which the body will appear to have the moſt uniform motion; for in this caſe the point F will be in the middle of the figure LpDl, which is the equant for the motion about that point; ſo that the body will appear to move about the centre F as ſwift when it is in its ſloweſt motion in the remoter apſis A as it does when it is in its ſwifteſt motion in the neareſt apſis P.

For, by the conſtruction, when the body is at A, the ray of the equant FL is a mean proportional between AF and AS; and when the body is at P, the ray of the equant Fl is a mean proportional between the two diſtances PS and PF, which are reſpectively equal to the former.

And in like manner, in an orbit of any other given form, a place may be found about which the motion is moſt regular.

If what has been ſaid be applied to the caſe of a body revolving in an elliptic orbit, and deſcribing equal areas in equal times about one of the foci, as is the caſe of a planet about the ſun, and a ſecondary planet about the primary one, it will ſerve to ſhew the foundation of the ſeveral hypotheſes and rules which have been invented by the modern aſtronomers for the equating of ſuch motions, and likewiſe ſhew how far each of them are deficient or imperfect.

For if the ellipſis ADP be the orbit of a planet deſcribing equal areas about the ſun in the focus S, the other focus, ſuppoſe F, will be the place about which the motion is moſt regular, from what has been already ſaid; that focus being at the ſame diſtance from the aphelion A as the ſun at S is from the perihelion P. And, by the conſtruction, each ray (Fp) of the equant will always be a mean proportional between FR and RS, the two diſtances of the planet from the two foci, in that place where the ray Fp is taken; for the rays SR and RF, making equal angles with the tangent at R, by the property of the ellipſis, are in the proportion of the perpendiculars from S and F, let fall on thoſe tangents. And, therefore, Fp being to FR in the ſubduplicate proportion of SR to FR, it will be a mean proportional between thoſe diſtances.

1. Hence when the planet is in the aphelion A, or perihelion P, the rays of the equant FL and Fl are the ſhorteſt, each being equal to CD, the leſſer ſemi-axis of the orbit; for, by the property of the ellipſis, the rectangle of the extreme diſtances from the focus is equal to the ſquare of the leſſer ſemi-axis.

2. When the planet is at its mean diſtance from the ſun in D or d, the extremities of the leſſer axis, the equant cuts the orbit in the ſame place; the rays of the equant being then the longeſt, being each equal to the greater ſemi-axis CA. For in thoſe points of the orbit the diſtances from the foci and the mean proportional are the ſame.

From which form of the equant it appears,

1. That the velocity of the revolution about the focus F diminiſhes in the motion of the planet from the aphelion or perihelion to the mean diſtance, and increaſes in paſſing from the mean diſtance to the perihelion or aphelion; for the rays of the equant increaſe in the firſt caſe, and diminiſh in the latter; and the velocity of revolution increaſes in the duplicate proportion, as the rays diminiſh.

2. In any place of the orbit, ſuppoſe R, the velocity of the revolution about the focus F is in proportion to the mean velocity, as the rectangle of the ſemi-axes of the orbit CD and CA to the rectangle of the focal diſtances RF and RS; for the equant and the orbit, being figures of the ſame area, are each equal to a circle whoſe radius is a mean proportional between the two ſemi-axes CD and CA. But the mean motion about the focus F is in thoſe places where the ſaid circle cuts the equant; and in other places the velocity of the revolution is reciprocally as the ſquare of the diſtance, that is, reciprocally as the rectangle of the focal diſtances RF and RS.

3. So that the planet is in its mean velocity of revolution about the focus F in four places of the orbit, that is, where the rectangle of the focal diſtances is equal to the rectangle of the ſemi-axes; which places in orbits nearly circular, ſuch as thoſe of the planets, are about 45 degrees from the aphelion or perihelion; but may be aſſigned in general, if need be, by taking a point in the orbit, ſuppoſe R, whoſe neareſt diſtance from the leſſer axis of the orbit CD is to the longer ſemi-axis CA in the ſubduplicate proportion of the longer axis to the ſum of the two axes; as may be eaſily proved.

What has been ſaid may be enough to ſhew the form of the equant, and the manner of the motion about the upper ſocus in general; but the preciſe determination of the inequality of the motion requires the knowledge of the quadrature of the ſeveral ſectors of the equant, or, at leaſt, if any other method be taken, of that which is equivalent to ſuch a quadrature.

There are divers methods for ſhewing the relation between the mean and true motion of a planet round the ſun, or round the other focus, ſome more exact than others; but the following ſeems the moſt proper for exhibiting in one view all the ſeveral hypotheſes and rules which are in common uſe in the modern aſtronomy, whereby it may eaſily appear how far they agree or differ from each other, and how much each of them errs from the preciſe determination of the motion, according to the true law of an equal deſcription of areas about the ſun.

Upon the centre F deſcribe the ellipſis LNl, equal and ſimilar to the elliptic orbit ADP, but having its axes FN and FL contrarily poſited, that is, the ſhorter axis LF *lying* in the longer axis of the orbit AP, and the longer axis FN parallel to the ſhorter CD. Let the focus of the ſaid ellipſis be in f; and ſuppoſe two other ellipſes LBl and Lfl to be drawn upon the common axis Ll, one paſſing through the point B, where the perpendicular FN interſects the orbit, and the other through the focus f. Let the line FR, revolving with the planet in the orbit, be indefinitely produced, till it interſect the firſt ellipſis LNl (which was ſimilar to the orbit) in Q, the equant in p, and the ellipſis LBl (drawn through the interſection B) in K. From the point K let fall KH perpendicular to the line of apſides AP, and let it be produced till it interſect the firſt ellipſis LNl in O, and the ellipſis Lfl (paſſing through the focus f) in E; and, laſtly, in the ellipſis LNl let GM be an ordinate equal and parallel to EH. In which conſtruction it is to be noted that the ellipſes Lfl and LBl are ſuppoſed as drawn only to divide the line OKH in given proportions, that KH may be to OH as the *latus rectum* of the orbit to the tranſverſe axis; and that EH or GM, the baſe of the elliptic ſegment GLM, may be to OH as the diſtance of the foci to the tranſverſe axis.

Which being premiſed, it will be eaſy to prove that the ſector pFL in the equant, or, which is the ſame thing, the ſector RSA in the orbit, is equal to the curvilinear area OKFMG, that is, equal to the elliptic ſector QFL, deducting the ſegment LMG, and adding or ſubducting the trilinear ſpace QKO, according as the angle RFA is leſs or greater than a right angle. Wherein it is to be noted that theſe ſigns of addition and ſubduction are to be uſed in general, if the angle AFR is taken from the aphelion in the firſt ſemi-circle, but towards the aphelion in the latter ſemi-circle; but if the angle AFR be taken the ſame way throughout the whole revolution, as is the method in aſtronomical calculations, then the ſegment and the trilinear ſpace in the latter ſemi-circle muſt be taken with the contrary ſigns to what are laid down.

Hence it appears that the inequality in the motion of a planet about the upper focus F conſiſts of three parts.

I. The firſt and principal of which is the inequality in the alteration of the angle QFL in making equal areas in the ellipſis LNl (Fig. 3); for if a circle equal to the ellipſis be deſcribed upon the centre F, ſince the radius (being a mean proportional between the two ſemi-axes) will fall without the ellipſis about the line of apſides, and within it about the middle diſtances, the angle QFL, which is proportional to the area deſcribed in the circle, will therefore increaſe faſter about the line of apſides, and ſlower about the middle diſtances, in deſcribing equal areas in the ellipſis, than it ought to do in the hypotheſis of Biſhop *Ward*, who makes the planet revolve uniformly about the focus. The equation to rectify this inequality is determined by the following rule.

The tangent of the angle QFL is to the tangent of the angle in the circle including the ſame area as the longer axis of the ellipſis to the ſhorter axis; and the difference of the angles whoſe tangents are in this proportion is the equation; as is manifeſt from what was before ſaid on the properties of an elliptic equant. From the ſame it alſo follows, that,

1. The greateſt equation is an angle whoſe ſine is to the radius as the difference of the axes to their ſum, or, which is

the ſame thing, as the ſquare of the diſtance of the foci to the ſquare of half the ſum of the axes. So that in ellipſes nearly circular, of different eccentricities, this greateſt equation will vary nearly in the duplicate proportion of the eccentricity.

2. In ellipſes nearly circular, the equation at any given angle QFL is to the greateſt equation nearly as the ſine of the double of the given angle to the radius; which follows from hence, that the equation is the difference of two angles whoſe tangents are in a given proportion, and nearly equal.

3. This equation adds to the mean motion in the firſt and third quadrant of mean anomaly, and ſubducts in the ſecond and fourth; as will eaſily appear from that the line QF, in deſcribing equal areas in the ellipſes, makes the angle to the line of the apſides leſs acute than it would be in an uniform revolution.

This is the equation which is accounted for in the hypotheſis of *Bullialdus;* for he ſuppoſes the motion of the planet in its orbit to be ſo regulated about the upper focus, that the tangents of the angles, from the lines of apſides, ſhall always be to the tangents of the angles anſwering to the mean anomaly in the proportion of the ordinates in the ellipſis to the ordinates in the circle circumſcribed; which in effect is the ſame as if he had made the true equant for its motion about the focus F to be the ellipſis as above deſcribed.

The ſame equation is alſo uſed by Sir *Iſaac Newton* in his ſolution of the *Keplerian* problem, in the ſcholium to the 31ſt prop. of the 1ſt book, and is there deſigned by the letter V.

But ſince the true equant LDl coincides with the elliptic equant in the extremities of the ſhorter axis at L and l, and falls within the ſame at its interſection with the longer axis FN, it follows, that the motion of the planet in the ſemicircle about the aphelion is ſwifter than according to the hypotheſis of an equal deſcription of areas in the ellipſis LNl, and for the ſame reaſon ſlower in the other ſemi-circle about the perihelion; the velocity about the centre F being always reciprocally in the duplicate proportion of the diſtance.

Which leads to the ſecond part of the inequality of the motion about the focus.

II. The equation to rectify this inequality is an angle anſwering to the ſegment GLM; which angle is to be added to the mean anomaly, to make the area of the elliptic ſector QFL.

This angle or equation is determined by the following rule. Let R be an angle ſubtended by an arc equal in length to the radius of the circle, viz. 57,29578 degrees; and let A be an angle whoſe ſine is to the radius as GM, the baſe of the ſegment, to FN, the ſemi-tranſverſe axis; alſo let B be an arc in proportion to R, as the ſine of the double of the angle A to the radius: then the equation for the ſegment will be equal to A — $\frac{1}{2}$B.

This equation is at its maximum when the angle LFQ is a right angle; the baſe of the ſegment becoming equal to Ff, half the diſtance of the foci; and the angle A, being in this caſe half the angle FDS formed at the extremity of the leſſer axis, and ſubtended by FS, the diſtance of the foci; which is commonly called the greateſt equation of the centre. And, conſequently, the arc B, in this caſe, is to R as the ſine of the ſaid greateſt equation of the centre is to the radius. So that, according to this rule, for the meaſure of the ſegment, it will follow, that,

1. This greateſt equation is in proportion to the greateſt equation of *Bullialdus*, as found in the preceding article for the elliptic equant, nearly as three times the tranſverſe axis to eight times the diſtance of the foci. Or, otherwiſe, the greateſt equation is to the angle deſigned by R as twice the cube of the diſtance between the foci to three times the cube of the tranſverſe axis. Either of which rules may be derived from the true angle, as before determined; or by taking $\frac{2}{3}$ of the rectangle of GM and LM, the baſe and height of the ſegment, for the meaſure of that ſegment.

So that in elliptic orbits nearly circular this greateſt equation for the ſegment is in the triplicate proportion of the eccentricity.

2. This equation at any given angle QFL is to the greateſt equation in the triplicate proportion of the ordinate OH to the ſemi-tranſverſe; that is, nearly as the cube of the ſine of the mean anomaly joined to the double of *Bullialdus*'s equation to the cube of the radius. For the ſegment GML, which is proportional to the equation, is in the triplicate proportion of its baſe nearly; and the baſe is proportional to the ordinate OH, by the conſtruction.

But the ordinate OH (in a circle deſcribed upon the radius FN) becomes the ſine of an angle whoſe tangent is to the tangent of the angle QFL in the proportion of the tranſverſe axis to the conjugate; but the tangent of the ſame angle QFL is to the tangent of the mean motion anſwering to the area of the elliptic equant QFL in the ſame proportion. So that the ordinate OH is to the ſine of that angle of mean motion in the duplicate of the ſaid proportion; and conſequently the ordinate OH, in the circle on the radius FN, is the ſine of an angle nearly equal to the mean anomaly joined to the double of *Bullialdus*'s equation.

3. This equation adds to the mean motion in paſſing from the aphelion to the perihelion, and ſubducts in paſſing from the perihelion to the aphelion; as is evident from the tranſit of the point of interſection E round the periphery of the ellipſis Lfl.

In Sir *Iſaac Newton*'s rule (in the before-cited ſcholium to the 31ſt prop. 1ſt book) the angle X anſwers to this equation for the ſegment; excepting that it is there taken in the triplicate proportion of the ſine of the mean anomaly, inſtead of the triplicate proportion of the ordinate OH. The error of this rule makes,

III. The third part of the inequality, anſwering to the trilinear ſpace OKQ, being the difference of the elliptic ſector OFQ and the triangle OFK.

The ſector OQF is proportional to an angle which is the difference of two angles whoſe tangents are in the given proportion of the ſemi-latus rectum FB and the ſemi-tranſverſe FN, or in the duplicate proportion of the leſſer axis to the axis of the orbit. So that this ſector, when at a maximum, is as an angle whoſe ſine is to the radius as the difference of the *latus rectum,* and tranſverſe to their ſum; or as the difference of the ſquares of the ſemi-axes to their ſum.

The triangle OFK is proportional to the rectangle of the co-ordinates OH and HF; that is, as the rectangle of the ſine OH and its coſine, in the circle on the radius FN; or as the ſine of the double of that angle whoſe ſine is OH; that is, the double of the angle whoſe tangent is to the tangent of the angle QFL in the given ratio of the greater to the leſſer axis; or whoſe tangent is the tangent of the angle of mean motion anſwering to the elliptic ſector QFL in the duplicate of the ſaid ratio. But this triangle OFK, when at a maximum, makes an angle of mean motion which is to the angle called R as BN, half the difference between the *latus rectum* and tranſverſe axis, is to the double of the tranſverſe axis.

So that the ſector or triangle in orbits nearly circular is always nearly equal to the double of *Bullialdus*'s equation.

The triangle and ſector being thus determined, the equation for the trilinear ſpace is accordingly determined. From what has been ſaid, it appears, that,

1. This equation for the trilinear ſpace OKQ is to that for the triangle OKF in a ratio compounded of BN, the difference between the ſemi-tranſverſe and ſemi-latus rectum to the ſemi-latus rectum, and of the duplicate proportion of the ſine OH to the radius; or OKQ is to OKF in a proportion compounded of the duplicate proportion of the diſtance of the foci to the ſquare of the leſſer axis, and the duplicate proportion of the ſine OH to the radius; for the trilinear figure OKQ and the triangle OKF are nearly as OK and KH, which are in that proportion; and conſequently it holds in this proportion to the double of *Bullialdus*'s equation.

2. This equation, in different angles, is as the content under the ſine complement and the cube of the ſine; for the triangle OKF is as the rectangle of the ſine and the ſine complement.

3. It is at a maximum at an angle whoſe ſine complement is to the radius as the ſquare of the greater axis is to the

ſum of the ſquares of the two axes; which in orbits nearly circular is about 60 degrees of mean anomaly.

4. In orbits of different eccentricities it increaſes in the quadruplicate proportion of the eccentricity.

5. It obſerves the contrary ſigns to that for the elliptic equant, called *Bullialdus's* equation; ſubducting from the mean motion in the firſt and third quadrants, and adding in the ſecond and fourth, if the motion is reckoned from the aphelion.

The uſe of theſe equations in finding the place of a planet from the upper focus will appear from the following rules, which are eaſily proved from what has been ſaid.

Let t be equal to CA the ſemi-tranſverſe, c equal to FC the diſtance of the centre from the focus, b equal to CD the ſemi-conjugate, and R an angle ſubtended by an arc equal to the radius, viz. $57^\circ\ 17'\ 44''\ 48'''$, or 57,2957795 degrees. Take an angle $T = \frac{cc}{2tt} R$; $E = \frac{b}{2t} T$; $S = \frac{4c}{3b} T$.

The angle T will be the greateſt equation for the triangle OFK; the angle S will be the greateſt equation for the ſegment LMG; and the angle E will be the greateſt equation for the area OKFL. Which greateſt equations being found, the equations at any angle of mean anomaly will be determined by the following rules.

Let M be the mean anomaly; and let τ be to T as the ſine of the angle 2M to the radius: in which proportion, as alſo in the following, there is no need of any great exactneſs, it being ſufficient to take the proportions in round numbers.

Take e to E as the ſine of $2M \pm 2\tau$ to the radius; and s to S as the cube of the ſine of $M \pm \tau$ to the cube of the radius.

Then the angle QFL is equal to $M + e + s$, in the firſt quadrant LN; or $M - e + s$, in the ſecond quadrant Nl; or $M + e - s$, in the third quadrant: or $M - e - s$, in the fourth quadrant.

Note, That the ſmall equation τ is always of the ſame ſine with the equation e; and in the caſe of the planets always near the double of that equation.

The angle RFA at the upper focus F being known, the angle RSA at the ſun in the other focus is found by the common rule of Biſhop *Ward;* viz. the tangent of half the angle RSA is to be to the tangent of half the angle RFA always in the given proportion of the perihelion diſtance SP to the aphelion diſtance SA. How theſe equations are in the ſeveral eccentricities of the moon's orbit, will appear by the following table.

Eccentr.	E.	S.
	′ ″	″
0.040	1.23	09
0.045	1.45	13
0.050	2.09	17
0.055	2.36	23
0.060	3.06	30
0.065	3.38	38
0.070	4.14	47

To add one example: ſuppoſe the eccentricity 0.060, the mean anomaly 30°. The ſine of the double of the mean anomaly, that is, the ſine of 60, is to the radius nearly as 87 to 100; whence, if the equation $E = 3' \, 06''$ be divided in that proportion, it will produce 2′ 40″ nearly for the equation e: the ſine of M is, in this caſe, equal to $\frac{1}{2}$ the radius, the cube is $\frac{1}{8}$ of the cube of the radius; whence if the equation $S = 30''$ be divided in the ſame proportion, it will produce near 4″ for the equation s. Therefore the angle RFA, which is $M + e + s$, will be 30° 2′ 44″; and the half is 15° 1′ 22″: wherefore if the tangent of this angle be diminiſhed in the proportion of 1.06, the aphelion diſtance, to 94, the perihelion diſtance, it will produce the tangent of 13° 23′ 13″; the double of which, 26° 46′ 26″, is the true anomaly or angle at the ſun RSA. And conſequently the equation of the centre is 3° 13′ 34″ to be ſubducted, at 30 degrees mean anomaly.

When the place of a planet is found by this or any other method, the place may be corrected to any degree of exactneſs by the common property of the equant, viz. that the rays are reciprocally in the duplicate proportion of the velocity about the centre; for in this caſe, if there be a difference between the mean motion belonging to the angle aſſumed at the upper focus and the given mean motion, the error of the angle aſſumed is to the difference as the rectangle of the ſemi-axes to the rectangle of the diſtances from the foci. But in orbits like thoſe of the planets the rules as they are delivered above are ſufficient of themſelves, without farther correction.

POSTSCRIPT.

UPON reviewing theſe few ſheets after they were printed off, which happened a little ſooner than I expected, I fear the apology I have offered for delivering the propoſitions relating to the moon's motion in this rude manner, without giving any proof of them, or ſo much as mentioning the fundamental principles of their demonſtration, will ſcarcely paſs as a ſatisfactory one; eſpecially ſince there are among theſe propoſitions ſome which, I am apt to think, cannot eaſily be proved to be either true or falſe by any methods which are now in common uſe.

Wherefore to render ſome ſatisfaction in this article, I ſhall add a few words concerning the principles from whence theſe propoſitions and others of the like nature are derived; and alſo take the opportunity to ſubjoin a few remarks which ought to have been made in their proper places.

Firſt, There is a law of motion, which holds in the caſe where a body is deflected by two forces tending conſtantly to two fixed points;

Which is, *That the body, in ſuch a caſe, will deſcribe, by lines drawn from the two fixed points, equal ſolids in equal times about the line joining the ſaid fixed points.*

The law of *Kepler,* that bodies deſcribe equal areas in equal times about the centre of their revolution, is the only general principle in the modern doctrine of centripetal forces.

But ſince this law, as Sir *Iſaac Newton* has proved, cannot hold, whenever a body has a gravity or force to any other than one and the ſame point, there ſeems to be wanting ſome ſuch law as I have here laid down, that may ſerve to explain the motions of the moon and ſatellites, which have a gravity towards two different centres.

It follows, as a corollary to the law here laid down, that if a body, gravitating towards two fixed centres, be ſuppoſed, for given ſmall intervals of time, as moving in a plane paſſing through one of the fixed centres, the inclination of the ſaid plane to the line joining the centres will vary according to the area deſcribed; that is, if the area be greater, the inclination will be leſs; and if the area be leſs, the inclination will be greater, in order to make the ſolids equal.

This corollary, when rightly applied, will ſerve to explain the variation of the inclination of the plane of the moon's orbit to the plane of the ecliptic.

And how extremely difficult it is to compute the variation of the inclination in any particular caſe, without the knowledge of ſome ſuch principle as this is, will beſt appear, if any one conſider the intricacy of the calculations uſed in the corollaries to the 34th prop. of the third book of the *Principia*, in order to ſtate the greateſt quantity of variation in that month when the line of the nodes is in quadrature with the ſun, and that only in particular numbers, whereby it is determined to be 2′ 43″.

Whereas, there is a plain and general rule in this caſe, which follows from what is laid down, though not immediately; namely, that the greateſt variation in the ſaid poſition of the moon's orbit is to the mean inclination of the plane as the difference of the greateſt and leaſt areas deſcribed in the ſame time by the moon about the earth, when in the conjunction and in the quarters, to the mean area.

Wherefore if S be to L as the ſun's period to the moon's period, the greateſt area is to the leaſt as $\sqrt{SS + 3LL}$ to S, or as $S + \frac{3LL}{2S}$ to S nearly, by what is ſaid on this article in the 208th page. So that the difference of areas is to the mean area as $\frac{3}{2}LL$ to $SS + \frac{3}{4}LL$; and in the ſame proportion is the greateſt variation of the inclination of the plane in this month to the mean inclination, which agrees nearly with Sir *Iſaac*'s computation.

Secondly, There is a general method for aſſigning the laws of the motion of a body to and from the centre, abſtractly conſidered, from its motion about the centre.

The motion to and from the centre is called by *Kepler* a libratory motion; the knowledge of which ſeems abſolutely requiſite to define the laws of the revolution of a body, in reſpect of the apſides of its orbit.

For the revolution of a body, from apſis to apſis, is performed in the time of the whole libratory motion; the apſides of the orbit being the extreme points wherein the libratory motion ceaſes.

So that, according to this method, the motion of a body round the centre is not conſidered as a continued deflection from a ſtraight line, but as a motion compounded of a circulatory motion round the centre, and a rectilinear motion to or from the centre.

Each of which motions require a proper equant. Of the equant for the motion round the centre I have already given ſeveral examples; and in the caſe of all motions which are governed by a gravity or force tending to a fixed point, the real orbit in which the body moves is the equant for this motion. In all other caſes it is a different figure

The equant for the libratory motion is a curve line figure, the areas of which ſerve to ſhew the time wherein the ſeveral ſpaces of the libration are performed.

Which figure is to be determined by knowing the law of the gravity to the centre; for the libratory force to accelerate or retard the motion to or from the centre is the difference between the gravity of the body to the centre and the centrifugal force ariſing from the circulatory motion. But the latter is always under one rule; for in all revolutions round a centre, in any curve line, whether deſcribed by a centripetal force or not, the centrifugal force is directly in the duplicate proportion of the area deſcribed in a given ſmall time, and reciprocally in the triplicate proportion of the diſtance; which is an immediate conſequence of a known propoſition of Mr. *Huygens*. The like proportion alſo holds as to the centripetal force in all circular motions, from a known propoſition of Sir *Iſaac Newton*. But what is true of the centripetal force in circles is univerſally true of the other force in orbits of any form.

So that by knowing the gravity of the body, ſince the other force is always known, the difference, which is the abſolute force to move the body to or from the centre, will be known; and from thence the velocity of the motion, and the ſpace deſcribed in any given time, may be found, and the equant deſcribed. Theſe hints may be ſufficient to ſhew what the method is.

To add an example. If the gravity be reciprocally as the ſquare of the diſtance, the equant for the libratory motion will be found to be an ellipſis ſimilar to the orbit, whoſe longer axis is the double of the eccentricity: the centre of the libratory motion, that is, the place where it is ſwifteſt, will be in the focus; the time of the libration through the ſeveral ſpaces is to be meaſured by ſectors of the ſaid ellipſis, ſimilar to thoſe deſcribed by the body round the focus of the orbit; and the period of the libratory motion will be the ſame with the period of the revolution.

In any other law of gravity, the equant for the libratory motion will either be of a form different from the orbit, or, if it be of the ſame form, it muſt not be ſimilarly divided.

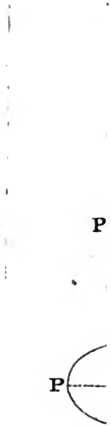
I may juſt mention that the equant for the libratory motion, in the caſe of the moon, is a curve of the third kind, or whoſe equation is of four dimenſions; but is to be deſcribed by an ellipſis, the centre of the libration not being in the focus.

From this method of reſolving the motion it will not be difficult to ſhew the general cauſes of the alteration of the eccentricity and inequality in the motion of the apogee; for when the line of apſides is moving towards the ſun, it may be eaſily ſhewn, that, ſince the external force in the apſides is then centrifugal, it will contribute to lengthen the ſpace and time of the libration: by lengthening the ſpace, it increaſes the eccentricity; and by lengthening the time of the libration, it protracts the time of the revolution to the apſis, and cauſes what is improperly called a motion of the apſis forward. But when the line of apſides is moving to the quadratures, the external force in the apſides is at that time centripetal, which will contribute to ſhorten the ſpace and time of libration; and by ſhortening the ſpace will thereby leſſen the eccentricity, and by ſhortening the time of libration will thereby contract the time of the revolution to the apſis, and cauſe what is improperly called a retrograde motion of the apſis.

I ſhall only add a few remarks, which ought to have been made in their proper places.

As to the motion of the moon in the elliptic epicycle (p. 198), it ſhould have been mentioned, that there is no need of any accurate and perfect deſcription of the curve called an ellipſis, it being only to ſhew the elongation of the moon from the centre of the epicycle, which doth not require any ſuch accurate deſcription.

It ſhould have been ſaid (Fig. 1), that when the moon is in any place of its orbit, ſuppoſe ſomewhere at N, in that half of the orbit which is next the ſun, it then being nearer the ſun than the earth, has thereby a greater gravity to the ſun than the earth: which exceſs of gravity, according to Sir *Iſaac Newton*'s method, conſiſts of two parts; one acting in the line NV, parallel to that which joins the earth and ſun, and the other acting in the line VB directed to the earth; and theſe two forces, being compounded into one, make a force directed in the line NB, which is in proportion to the force of gravity as that line NB is to TB nearly. Wherefore as there is a force conſtantly impelling the moon ſomewhere towards the point B, this force is ſuppoſed to inflect the motion of the moon into a curve line about that point; for the ſame reaſon as the gravity of it to the earth is ſuppoſed to inflect its motion into a curve line about the earth: not that the moon can actually have ſo many diſtinct motions, but the one ſimple motion of the moon round the ſun is ſuppoſed to ariſe from a compoſition of theſe ſeveral motions.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P

In the laſt article on the ſmall annual equations (page 213), theſe rules ought to have been added.

Let Æ be the equation of the ſun's centre; P the mean periodical time of the node or apogee; S the mean ſynodical time of the ſun's revolution to the node or apogee: then will $\frac{3S}{2P}$ Æ be the annual equation of the node or apogee, according as S and P are expounded.

The like rule will ſerve for the annual equation of the moon's mean motion. If S be put for the ſun's period, P for the mean ſynodical period of the moon to the ſun, and L for the moon's period to the ſtars, the annual equation of the moon's mean motion will be $\frac{3LL}{2PS}$ Æ.

According to theſe rules, when expounded, the equation for the node will be found to be always in proportion to the equation of the ſun's centre, nearly as 1 to 13;

The equation of the apogee to the equation of the ſun's centre as 10 to 53;

And the equation of the moon's mean motion to the ſame as 8 to 77.

It may be throughout obſerved, that the propoſitions are in general terms, ſo as to ſerve, *mutatis mutandis*, for any other ſatellite as well as the moon.

There might have been ſeveral other obſervations and remarks made in many other places, had there been ſufficient time for it; but, perhaps, what I have already ſaid may be too much, conſidering the manner in which it is delivered.

FINIS.

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[illegible]

[illegible] ought to have been [illegible].

[illegible] the equation [illegible]

[illegible] time of the [illegible]

[illegible] sun's revolution to the [illegible]

the annual equation of [illegible]

and P are expounded.

[illegible] rule will serve for the [illegible] mean motion. If S [illegible] synodical period of the [illegible] this period to the [illegible]

[illegible] mean motion will [illegible]

[illegible] according to these rules [illegible] [illegible] will be found to be [illegible] [illegible] of the sun's [illegible] the equation of the apog[illegible] [illegible] are as 16 to 33; and the equation of [illegible] [illegible] to 77. [illegible] may be [illegible] [illegible] her satellite [illegible] [illegible]

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